Name______________________________ Period __________ Date _______________

GRADE 7
RESOURCE GUIDE
PART 2

MATHLINKS: GRADE 7
RESOURCE GUIDE: PART 2

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STANDARDS FOR MATHEMATICAL PRACTICE

In addition to the mathematical topics you will learn about in this course, your teacher will help you become better at what are called the Mathematical Practices. The Standards for Mathematical Practice describe a variety of processes and strategies to help you to be more mathematically proficient and fluent students.

One way to think about the practices is in groupings.

HABITS OF MIND

MP1 Make sense of problems and persevere in solving them
MP6 Attend to precision

REASONING AND EXPLAINING

MP2 Reason abstractly and quantitatively
MP3 Construct viable arguments and critique the reasoning of others

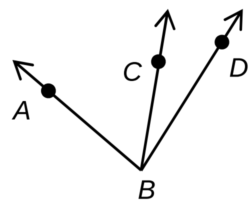
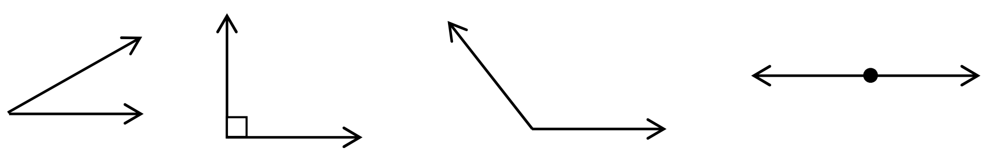
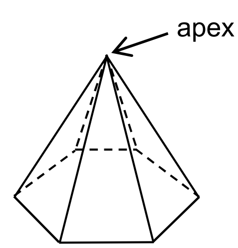
MODELING AND USING TOOLS

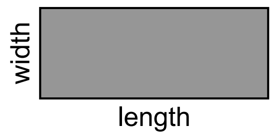
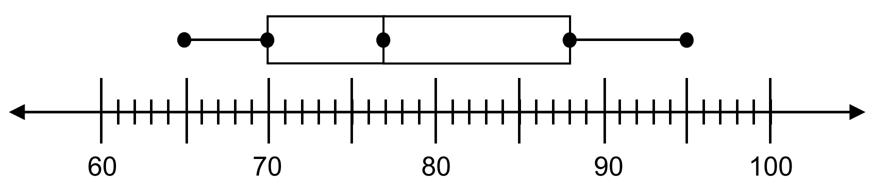
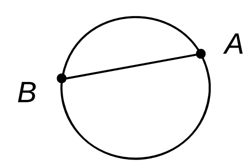
MP4 Model with mathematics
MP5 Use appropriate tools strategically

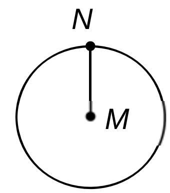
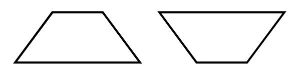
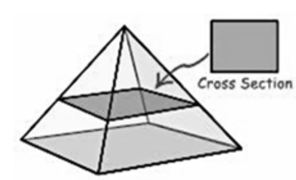
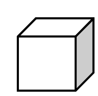
SEEING STRUCTURE AND GENERALIZING

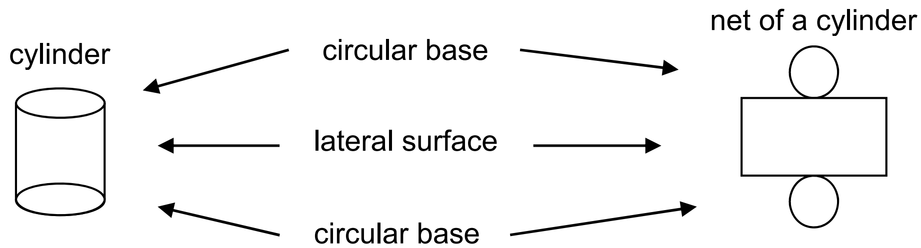
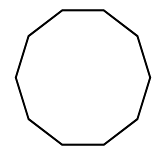
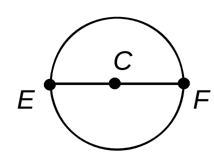
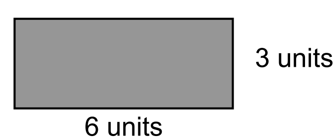
MP7 Look for and make use of structure
MP8 Look for and make use of repeated reasoning

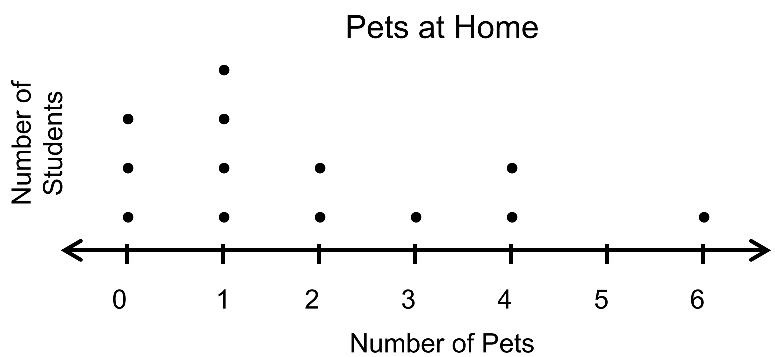
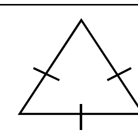
WORD BANK

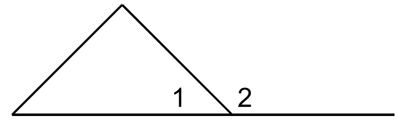
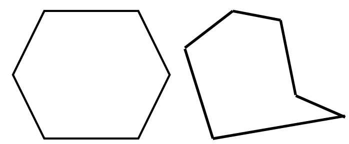
Word or Phrase	Definition
acute angle	An <u>acute angle</u> is an angle whose measure is less than 90°. See <u>angle</u>.
adjacent angles	Two angles are <u>adjacent</u> if they have the same vertex and share a common ray, and they lie on opposite sides of the common ray. Example: $\angle ABC$ and $\angle CBD$ are adjacent angles. 
angle	An <u>angle</u> is a geometric shape formed by two (distinct) rays that share a common endpoint (the <u>vertex</u> of the angle). To each angle is assigned a <u>degree measure</u>, which is a measure of the size of the angle and which is between 0 and 180 degrees. • An <u>acute angle</u> is an angle whose measure is less than 90°. • A <u>right angle</u> is an angle whose measure is exactly 90°. • An <u>obtuse angle</u> is an angle whose measure is between 90° and 180°. • A <u>straight angle</u> is an angle whose measure is 180°. The sides of a straight angle are opposite rays that form a straight line.  acute angle right angle obtuse angle straight angle
apex	The <u>apex</u> of a convex polyhedron with a single base is the vertex at the greatest distance from the plane of the base. Example: If the polyhedron is viewed as sitting on its base, the apex is its highest point. Example: The apex of a pyramid is the vertex that does not belong to the base. 

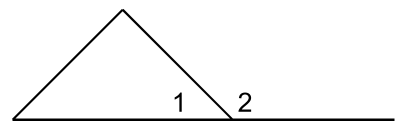
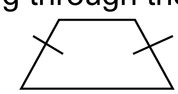
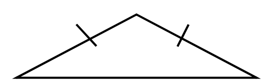
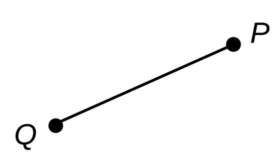
approximation	An <u>approximation</u> is an inexact result or estimate that is adequate for the purpose at hand. Example: The approximation 3.14 to π = 3.14159… is accurate enough for most purposes.
area	The <u>area</u> of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The <u>area of a rectangle</u> is the product of its length and its width.  Area = (length) × (width) Example: If a rectangle has a length of 12 inches and a width of 5 inches, its area is $5 \times 12 = 60$ square inches.
box plot	A <u>box plot</u>, or <u>box-and-whiskers plot</u>, is a graphical representation of the five-number summary of a data set. There are several variants of box plots. In one of these, the minimum, maximum, median, and quartiles of the data set are indicated by dots on a number line, a box from the first quartile to the third quartile encloses the middle half of the data set, and whiskers reach out from the box to the minimum and maximum. See <u>five-number summary</u>. Example: A box-and-whiskers plot for test scores ranging from a minimum score *min* = 65 to a maximum score *max* = 95, with median score M =77, first quartile Q_1 = 70, and third quartile Q_3 = 88: 
center of a circle	See <u>circle</u>.
chord	A <u>chord</u> of a circle is a line segment whose endpoints lie on the circle. If the chord passes through the center of the circle, it is a <u>diameter</u> of the circle. Example: The segment from A to B is a chord. 

circle	A circle is a closed curve in a plane consisting of all points at a fixed distance (the radius) from a specified point (the center). Example: The center is at M and the radius is the length of the line segment from M to N. 
circumference	The circumference of a circle is the length of the circle, that is, the distance around it. The circumference of a circle of radius r is $C = 2\pi r$. See circle.
complementary angles	Two angles are complementary if the sum of their measures is 90°. Example: Two angles that measure 30° and 60° are complementary.
congruent figures	Two figures are congruent if one can be moved to exactly cover the other by translations (slides), rotations (turns), and reflections (flips). Congruent figures have exactly the same size and shape. 
counterexample	A counterexample to a mathematical statement is an example for which the statement is false. Example: The statement $x^2 \leq 0$ is false. A counterexample is given by $x = 1$.
cross-multiplication property	The cross-multiplication property states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. Example: From $\frac{2}{3} = \frac{8}{12}$ we have $3 \bullet 8 = 2 \bullet 12$.
cross section	The intersection of a solid figure with a plane is a cross section of the figure. 
cube	A cube is a six-sided polyhedron in which all the faces are squares. See right rectangular prism. 

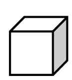
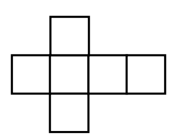
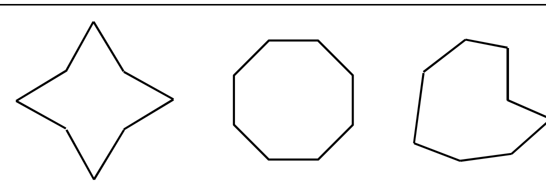
cylinder	A (right circular) <u>cylinder</u> is a figure in three-dimensional space that has two parallel circular bases. These circles are connected by a curved surface, called the <u>lateral surface</u>, which is a “rolled up” rectangle. Example: Most soup cans have the shape of a right circular cylinder. 
decagon	A <u>decagon</u> is a polygon that has ten sides. 
decrease in a quantity	The <u>decrease in a quantity</u> is the original value minus the new value. The <u>percent decrease</u> in a quantity is the value of the ratio of the decrease to the original quantity, expressed as a percent. Example: Last year there were 200 students in the school. This year there are 178 students in the school. • The decrease in the number of students is 200 – 178 = 22. • Since $\frac{22}{200} = \frac{11}{100}$, the percent decrease is 11%.
diameter	A <u>diameter</u> of a circle is a line segment joining two points of the circle that passes through the center of the circle. Example: The line segment from *E* to *F* is a diameter. 
dimensions of a rectangle	The <u>dimensions of a rectangle</u> are its length and its width. Example: This rectangle’s dimensions are 6 units by 3 units. 

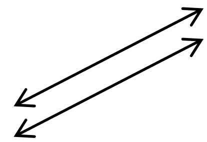
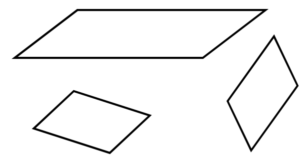
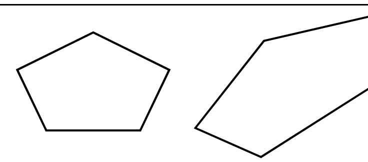
discount	The discount (or markdown) of an item is the decrease in the price of the item, that is, the original price of the item minus the new price. The percent discount is the percent decrease in the price of the item, that is, the value of the ratio of the decrease to the original value, expressed as a percent. Example: Last week, the price of an MP3 player was \$200. This week, the price is \$178. ● The discount is $200 - 178 = 22$. ● Since $\frac{22}{200} = \frac{11}{100}$, the percent discount is 11%.
dot plot	A dot plot is a graphical representation of a data set where the data values are represented by dots above a number line. See line plot. Example: The number of pets at homes of 13 different students are given by the data set {2, 3, 4, 1, 1, 0, 2, 1, 1, 4, 6, 0, 0}, with dot plot: 
equidistant	Two points P and Q are equidistant from a point C if the distance from P to C is equal to the distance from Q to C. Example: Points on a circle are equidistant from the center of the circle.
equilateral triangle	An equilateral triangle is a triangle whose three sides have equal length. See triangle. 
estimate	An estimate is an educated guess. Example: If the price of avocados is 89 cents each, and you wish to buy 4 avocados, a good estimate of the total cost might be 4 times 90 cents, or \$3.60.

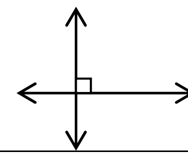
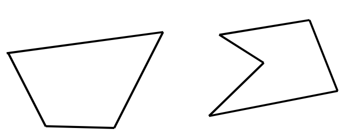
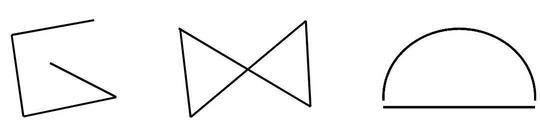
exterior angle	An <u>exterior angle</u> of a triangle is the angle between one side of the triangle and the straight-line extension of the adjacent side. Example: $\angle 1$ is an interior angle, and $\angle 2$ is an exterior angle of the triangle. 
five-number summary	The <u>five-number summary</u> of a data set consists of its minimum value *(min)*, first quartile Q_1, median M (or Q_2), third quartile Q_3, and maximum value *(max)*. The five-number summary is usually written in the form (min, Q_1, M, Q_3, max). Example: The five-number summary of the data set {1, 1, 1, 3, 5, 5, 6, 7, 23} is given by (min, Q_1, M, Q_3, max) = (1, 1, 5, 6.5, 23).
hexagon	A <u>hexagon</u> is a polygon that has six sides. 
horizontal	<u>Horizontal</u> refers to being in the same direction as the horizon. The horizontal direction is perpendicular to the force of gravity. Example: On a sheet of paper, typically horizontal is the direction that runs left to right. 
increase in a quantity	The <u>increase in a quantity</u> is the new value minus the original value. The <u>percent increase</u> in a quantity is the value of the ratio of the increase to the original quantity, expressed as a percent. Example: Last year there were 200 students in school. This year there are 208 students. • The increase in the number of students is 208 – 200 = 8. • Since $\frac{8}{200} = \frac{4}{100}$, the percent increase is 4%.
interest	<u>Interest</u> is an amount charged or paid for the use of money. Example: If \$10,000 was borrowed from a bank and the amount paid back to the bank was \$11,000, the interest charged by the bank was \$1,000.

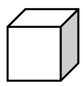
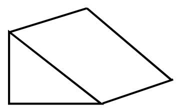
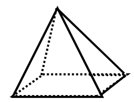
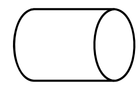
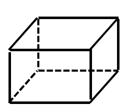
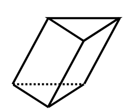
interest rate	The interest rate is the percentage of the principal paid per unit time for the use of the money. The annual interest rate is the percentage of the principal paid for the use of the money for one year. In many contexts, an interest rate is assumed to be an annual interest rate. Example: Interest on a one-year loan of \$10,000 at an annual interest rate of 10% is 10% of \$10,000, which is \$1,000.
interior angle	An interior angle of a triangle is the angle between two sides at the vertex where they meet. See triangle. Example: $\angle 1$ is an interior angle, and $\angle 2$ is an exterior angle of the triangle. 
interquartile range	The interquartile range (IQR) of a numerical data set is the distance between the first and third quartiles of the data set. The interquartile range is a measure of the variation of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is $Q_1 = 6$, the third quartile is $Q_3 = 15$, and the interquartile range is $\text{IQR} = Q_3 - Q_1 = 15 - 6 = 9$.
isosceles trapezoid	An isosceles trapezoid is a trapezoid with exactly one pair of parallel sides, in which the nonparallel sides are congruent. The line passing through the midpoints of the parallel sides of an isosceles trapezoid is a line of symmetry. See trapezoid. 
isosceles triangle	An isosceles triangle is a triangle that has at least two sides of equal length. 
lateral face	A lateral face of a solid figure such as a prism or cylinder refers to a face that is not a base. See prism, cylinder.
line plot	A line plot is a graphical representation of a data set where the data values are represented by marks, such as dots or X's, above a number line. See dot plot.
line segment	A line segment is a straight-line path joining two points. The line segment between two points P and Q consists of all points on the straight line through P and Q that lie between P and Q. The points P and Q are the endpoints of the line segment. 

markup	The markup on an item is the increase in the price of the item, that is, the new price of the item minus the original price. The percent markup is the percent increase in the price of the item. Example: Last week, the price of an MP3 player was \$200. This week, the price is \$208. • The markup is $208 - 200 = 8$. • Since $\frac{8}{200} = \frac{4}{100}$, the percent markup is 4%.
mean	The mean of a data set is the average of the values in the data set. The mean is calculated by adding the values in the data set and dividing by the number of data values. Example: For the data set {3, 3, 5, 6, 6}, the mean (average) is $\frac{3+3+5+6+6}{5} = 4.6$.
mean absolute deviation	The mean absolute deviation (MAD) is a measure of variation in a numerical data set, computed by adding the distances between each data value and the mean, and then dividing by the number of data values. Example: For the data set {3, 3, 5, 6, 6}, the mean is 4.6. The distances of the data points to the mean are 1.6, 1.6, 0.4, 1.4, and 1.4. The MAD is $\frac{\lvert 3-4.6\rvert + \lvert 3-4.6\rvert + \lvert 5-4.6\rvert + \lvert 6-4.6\rvert + \lvert 6-4.6\rvert}{5}$ $= \frac{1.6+1.6+0.4+1.4+1.4}{5} = 1.28$.
measure of center	A measure of center is a statistic describing the middle of a numerical data set. The mean, the median, and the mode are three commonly used measures of center. Example: For the data set {3, 3, 5, 6, 6}, the mean (average) is $\frac{(3+3+5+6+6)}{5} = 4.6$, and the median is 5. There are two modes, 3 and 6. Each of these numbers can be viewed as the "center" of the data set in some way.

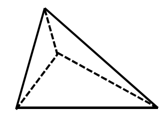
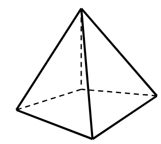
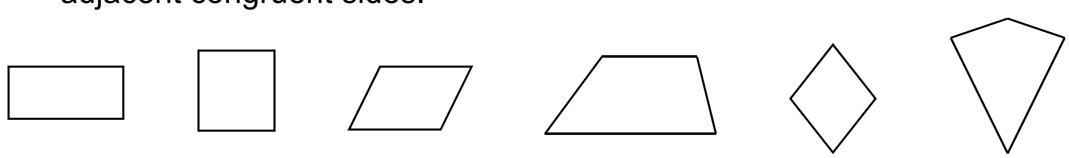
measure of spread	A measure of spread is a statistic describing the variability of a numerical data set. It describes how far the values in a data set are from the mean. Example: The standard deviation (SD or σ), the mean absolute deviation (MAD), and the interquartile range (IQR) are three measures of spread.
median	The median of a data set is the middle number in the data set, when the values are placed in order from least to greatest. If there is an even number of values in the data set, the median is taken to be the mean (average) of the two middle values. Example: The median of the data set {1, 1, 1, 3, 5, 5, 6, 7, 23} is 5, since the first 5 is the middle value. Example: The median of the data set {5, 6, 7, 23} is the mean (average) of the two middle numbers, $(6 + 7) \div 2 = 6.5$.
mode	The mode of a data set is the value (or values) that occurs most often. A data set may have more than one mode. Example: The mode of the data set {1, 1, 1, 3, 5, 6, 6, 7, 23} is 1, since the data value 1 occurs more frequently than any other data value.
net	A net for a three-dimensional figure is a two-dimensional pattern for the figure. When cut from a sheet of paper, a net forms one connected piece which can be folded and the edges joined to form the given figure. Example: cube  net of a cube 
obtuse angle	An obtuse angle is an angle whose measure is between 90° and 180°. See angle.
octagon	An octagon is a polygon that has eight sides. 

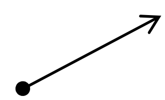
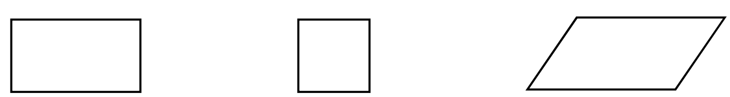
outlier	An outlier of a data set is a data value that is a striking deviation from the overall pattern of values in the data set. Example: For the data set {1, 1, 1, 3, 5, 6, 6, 7, 23}, the data value 23 is a potential outlier. It appears unusually large relative to other data values.
parallel	Two lines in a plane are parallel if they do not meet. Two line segments in a plane are parallel if the lines they lie on are parallel. 
parallelogram	A parallelogram is a quadrilateral in which opposite sides are parallel. In a parallelogram, opposite sides have equal length and opposite angles have equal measure. 
pentagon	A pentagon is a polygon that has five sides. 
percent	A percent is a number expressed in terms of the unit $1\% = \frac{1}{100}$. To convert a positive number to a percent, multiply the number by 100. To convert a percent to a number, divide the percent by 100. Example: Fifteen percent = $15\% = \frac{15}{100} = 0.15$ Example: $4 = 4 \times 100\% = 400\%$
percent decrease	The percent decrease in a quantity is the value of the ratio of the decrease to the original quantity, expressed as a percent. See decrease in a quantity.
percent increase	The percent increase in a quantity is the value of the ratio of the increase to the original quantity, expressed as a percent. See increase in a quantity.
percent of a number	A percent of a number is the product of the percent and the number. It represents the number of parts per 100 parts. Example: 15% of 300 is $\frac{15}{100} \bullet 300 = 45$. Example: If 45 out of 300 students are boys, then 15 out of every 100 students are boys, and 15% of the students are boys.

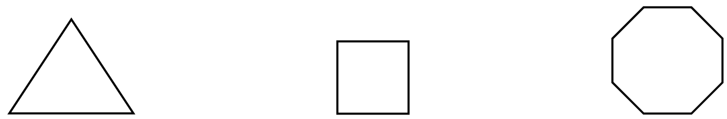
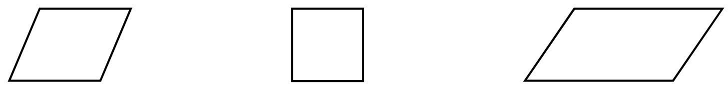
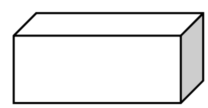
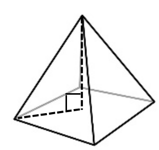
perimeter	The perimeter of a plane figure is the length of the boundary of the figure. Example: The perimeter of a square is four times its side-length. Example: The perimeter of a rectangle is twice the length plus twice the width. The perimeter of a circular disc is its circumference, which is π times its diameter.
perpendicular	Two lines are perpendicular if they intersect at right angles. See right angle. 
pi	Pi (written π) is the Greek letter used to denote the value of the ratio of the circumference of a circle to its diameter. Pi is an irrational number, with decimal representation $\pi = 3.14159\ldots$. The rational numbers 3.14 and $\frac{22}{7}$ are often used to approximate π.
plane	A plane refers to a flat two-dimensional surface that has no holes and that extends to infinity in all directions.
plane figure	A plane figure refers to a figure that lies in a plane.
point	A location on a line, on a plane, or in space, is referred to as a point. Example: In the coordinate plane, points are labeled as ordered pairs, such as (-2, 3).
polygon	A polygon is a special kind of figure in a plane made up of a chain of line segments laid end-to-end to enclose a region. Each endpoint of a segment of the polygon meets one other segment, otherwise the segments do not meet each other. The line segments are the sides (or edges) of the polygon, and the endpoints of the line segments are the vertices of the polygon. A polygon divides the plane into two regions, an “inside” and an “outside.” The region inside a polygon may also be referred to as a polygon.  polygons  not polygons

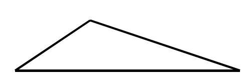
polyhedron	A polyhedron is a closed figure in three-dimensional space consisting of a finite number of polygons that are joined at their edges and that form the boundary of the enclosed solid figure. The polygons are the faces of the polyhedron, the edges of the polygons are the edges of the polyhedron, and the vertices of the polygons are the vertices of the polyhedron. A polyhedron divides space into two regions, an “inside” and an “outside.” The region inside a polyhedron may also be referred to as a polyhedron. Examples: A cube is a polyhedron. It has 6 faces, 12 edges, and 8 vertices. A cylinder is not a polyhedron.  polyhedron (cube)  polyhedron (triangular prism)  polyhedron (square pyramid)  not a polyhedron (cylinder)
population	The population is the entire group of individuals (objects or people) to which a statistical question refers. Example: If a survey is taken to investigate how many pets the students at Seaside School own, the population under study is the entire student body of Seaside School.
principal	Principal is an initial amount of money borrowed or deposited.
prism	A prism is a polyhedron in which two faces (the bases) are congruent parallel polygons, and the other faces (the lateral faces) are parallelograms. If the lateral faces are perpendicular to the bases, the prism is a right prism. Otherwise, the prism is an oblique prism. Example: A right rectangular prism is a right prism whose bases are rectangles. Example: A triangular prism is a prism whose bases are triangles.  right rectangular prism  oblique triangular prism

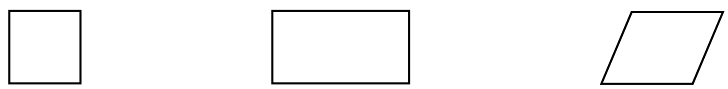
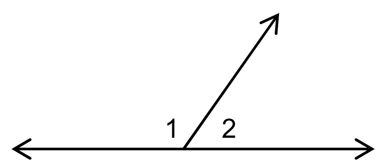
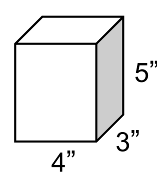
probability	The probability of an event is a measure of the likelihood of that event occurring. The probability $P(E)$ of an event E occurring satisfies $0 \le P(E) \le 1$. If the event E is certain to occur, then $P(E) = 1$. If the event E is impossible, then $P(E) = 0$. Example: When flipping a coin, the probability that it will land on heads is $\frac{1}{2} = 0.5 = 50\%$.
proportion	A proportion is an equation stating that the values of two ratios are equal. Example: The equation $\frac{3}{25} = \frac{12}{100}$ is a proportion. It asserts that the values of the ratios 3 : 25 and 12 : 100 are equal.
proportional	Two quantities are proportional if one is a multiple of the other. We say that y is proportional to x if $y = kx$, where k is the constant of proportionality. Example: If Bowser eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.
proportional relationship	Two variables are in a proportional relationship if the values of one are the same constant multiple of the values of the other. The constant is referred to as the constant of proportionality. Example: If Bowser eats 3 cups of kibble each day, then the number of cups of kibble and the number of days are in a proportional relationship.
protractor	A protractor is an instrument with degree measures marked on a circular arc used for measuring angles.

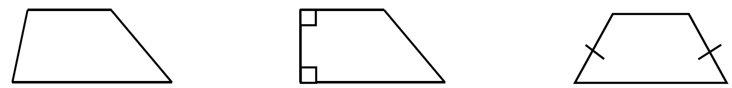
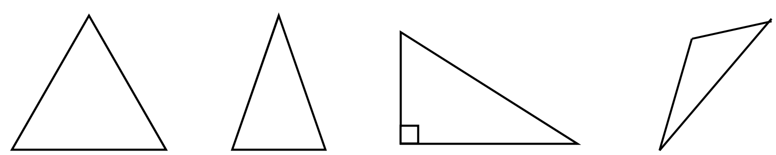
pyramid	A pyramid is a polyhedron in which one face (the base) is a polygon, and the other faces are triangles with a common vertex (the apex). Each edge of the base is the side of a triangular face with the opposite vertex at the apex. Example: A tetrahedron is a pyramid with a triangular base.  Example: A square pyramid is a pyramid with a square base. The Egyptian pyramids are square pyramids, as they have square bases. 
quadrilateral	A quadrilateral is a four-sided polygon. • A rectangle is a quadrilateral with four right angles. • A square is a quadrilateral with four congruent sides and four right angles. • A parallelogram is a quadrilateral in which opposite sides are parallel. • A trapezoid is a quadrilateral with at least one pair of parallel sides. • A rhombus is a quadrilateral whose four sides are congruent. • A kite is a quadrilateral whose four sides consist of two pairs of adjacent congruent sides.  rectangle square parallelogram trapezoid rhombus kite
radius	A radius of a circle is a line segment from the center of the circle to a point on the circle. The radius of a circle also refers to the length of that line segment. See circle.
range	The range of a numerical data set is the difference between the greatest and least values in the data set. Example: The range of the data set $\{1, 1, 1, 3, 5, 5, 6, 7, 23\}$ is 22, since $22 = 23 - 1$.
rate	See unit rate.

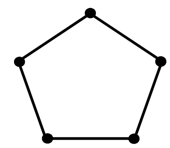
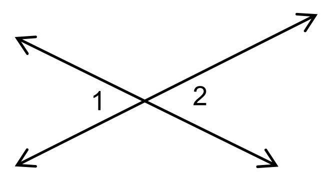
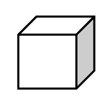
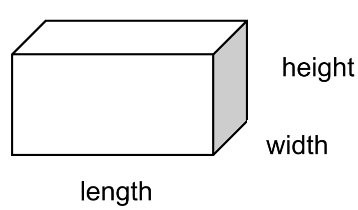
ratio	A <u>ratio</u> is a pair of nonnegative numbers, not both zero, in a specific order. The ratio of a to b is denoted by $a : b$ (read "a to b", or "a for every b"). The <u>value</u> of a ratio $a : b$, $b \neq 0$, is the quotient number $a \div b$. Example: The ratio of 3 to 2 is denoted by 3 : 2. The value of the ratio of 3 to 2 is $\frac{3}{2} = 1.5$.
rational number	A <u>rational number</u> is a number expressible in the form $\frac{m}{n}$, where m and n are integers, and $n \neq 0$. Example: $\frac{3}{5}$ is rational because it *is* a quotient of integers. Example: $2\frac{1}{3}$ and 0.7 are rational numbers because they *can be* expressed as quotients of integers, $2\frac{1}{3} = \frac{7}{3}$ and $0.7 = \frac{7}{10}$. Example: $\sqrt{2}$ and π are NOT rational numbers. They *cannot be* expressed as a quotient of integers.
ray	A <u>ray</u> is a half-line emanating from a point (the <u>vertex</u> of the ray). Example: The two sides of an angle are rays emanating from the vertex of the angle. 
rectangle	A <u>rectangle</u> is a quadrilateral with four right angles. In a rectangle, opposite sides are parallel and have equal length. Example: A square is a rectangle with four congruent sides.  rectangle square not a rectangle

regular polygon	A regular polygon is a polygon whose sides are all congruent, and whose angles are all congruent. The sides of a regular polygon have the same length, and the angles have the same angle measure. Example: A regular triangle is an equilateral triangle, and a regular quadrilateral is a square. A stop sign has the shape of a regular octagon. 
rhombus	A rhombus is a parallelogram whose sides have equal length. Example:  is a rhombus is a rhombus not a rhombus
right angle	A right angle is an angle that measures 90°. See angle.
right prism	A right prism is a prism whose lateral faces are perpendicular to the bases. See prism.
right rectangular prism	A right rectangular prism is a six-sided polyhedron in which all the faces are rectangles. The opposite faces of a right rectangular prism are parallel to each other. The distances between pairs of opposite faces are the length, width, and height of the right rectangular prism. Example: A rectangular box is a right rectangular prism. 
right rectangular pyramid	A right rectangular pyramid is a pyramid whose base is a rectangle, such that the line from the apex of the pyramid to the center of the base is perpendicular to the base. See pyramid.  Example: The Egyptian pyramids are right rectangular pyramids.
right trapezoid	A right trapezoid is a trapezoid with adjacent right angles. See trapezoid.

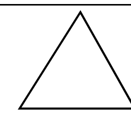
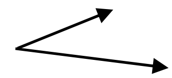
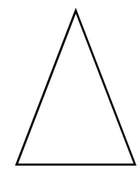
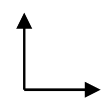
sample	A sample is a subset of the population that is examined in order to make inferences about the entire population. The sample size is the number of elements in the sample. Example: In order to estimate how many radios coming off the production line were defective, the plant manager selected a sample of 12 radios and tested them to see if they worked.
scale	In a scale drawing of a figure, the scale is the ratio of lengths in the drawing to lengths in the figure. Example: The blueprint of a house floorplan has a scale of 1 inch to 5 feet, or 1 in : 5 ft. Each inch on the blueprint represents 5 feet. Example: The map has a scale of 3 centimeters to 10 kilometers, or 3 cm : 10 km. Each 3 centimeters on the map represents 10 kilometers.
scale drawing	A scale drawing of a geometric figure is a drawing in which all lengths have been multiplied by the same scale factor. Example: A blueprint of a house floorplan is a scale drawing.
scale factor	A scale factor is a positive number which multiplies some quantity. Example: To make a scale drawing of a figure, we multiply all lengths by the same scale factor. If the scale factor is greater than 1, the figure is expanded, and if the scale factor is between 0 and 1, the figure is reduced in size.
scalene triangle	A scalene triangle is a triangle such that no two sides have the same length. See triangle. 
simple interest	Simple interest is interest paid only on the principal. Example: If you borrow \$100 for a period of time at a simple interest of 8%, then the interest will be $(\$100)\ (8\%) = (\$100)\left(\frac{8}{100}\right) = \$8.$

simulation	Simulation is the imitation of one process by means of another process. Example: We may simulate rolling a number cube by drawing a card blindfold from a group of six identical cards labeled one through six. Example: We may simulate the weather by means of computer models.
solid figure	A solid figure refers to a figure in three-dimensional space such as a prism or a cylinder.
square	A square is a rectangle whose sides have equal length. Example:  is a square not a square not a square
straight angle	A straight angle is an angle that measures 180°. See angle.
supplementary angles	Two angles are supplementary if the sum of their measures is 180°. Example: Angles 1 and 2 are supplementary because they determine a straight line, or 180°. 
surface area	The surface area of a three-dimensional figure is a measure of the size of the surface of the figure, expressed in square units. If the surface of the three-dimensional figure consists of two-dimensional polygons, the surface area is the sum of the areas of the polygons. Example: A rectangular box of length 3”, width 4”, and height 5” has surface area 94 square inches: $2(3 \bullet 4) + 2(3 \bullet 5) + 2(4 \bullet 5) = 94$. 

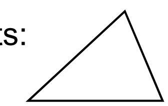
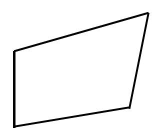
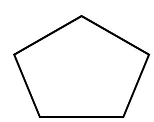
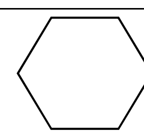
trapezoid	A trapezoid is a quadrilateral with a pair of parallel sides. An isosceles trapezoid is a trapezoid with exactly one pair of parallel sides, in which the nonparallel sides are congruent. A right trapezoid is a trapezoid with adjacent right angles.  trapezoid right trapezoid isosceles trapezoid
triangle	A triangle is a three-sided polygon. Triangles may be classified by their sides or their angles. If the three sides of the triangle have the same length, it is an equilateral triangle. If at least two sides have the same length, it is an isosceles triangle. If no two sides have the same length, it is a scalene triangle. If the three angles of a triangle have the same measure, it is an equiangular triangle. If all angles of the triangle are less than 90°, it is an acute triangle. If one of the angles of the triangle equals 90°, it is a right triangle. If one of the angles of the triangle is greater than 90°, it is an obtuse triangle.  classified by sides: equilateral, isosceles, scalene, scalene classified by angles: equiangular and acute, acute, right, obtuse
triangular prism	A triangular prism is a prism whose bases are triangles. See prism.
unit price	A unit price is a price for one unit of measure.
unit rate	The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. Example: The ratio of 40 miles each 5 hours has unit rate 8 miles per hour.

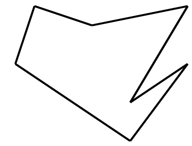
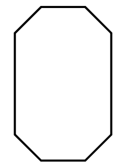
value of a ratio	The <u>value of the ratio</u> $a : b$ is the number $\frac{a}{b}$, $b \neq 0$. See <u>ratio</u>. Example: The value of the ratio $6 : 2$ is $\frac{6}{2} = 3$. Example: The value of the ratio of 3 to 2 is $\frac{3}{2} = 1.5$.
vertex	A <u>vertex</u> of a polygon is a point where two edges meet. See <u>polygon</u>. Example: A pentagon has five vertices. 
vertical	<u>Vertical</u> refers to being in the same direction as the force of gravity. The vertical direction is perpendicular to the horizontal direction. Example: On a sheet of paper, typically the vertical direction runs up and down. 
vertical angles	Two angles are <u>vertical angles</u> if they are opposite angles formed by a pair of intersecting lines. Example: $\angle 1$ and $\angle 2$ are vertical angles. 
volume	The <u>volume</u> of a three-dimensional figure is a measure of the size of the figure, expressed in cubic units. The volume of a right rectangular prism is the product of its length, width, and height. The volume of a union of non-overlapping figures is the sum of their volumes. Example: The volume of a cube of side length 3 units is $3 \bullet 3 \bullet 3 = 27$ cubic units. 
volume of a right rectangular prism	The <u>volume of a right rectangular prism</u> is the product of its length, width, and height. See <u>right rectangular prism</u>. Example:  Volume = (length) • (width) • (height)

GREEK AND LATIN WORD ROOTS

equi	“equal”: An equilateral triangle has three equal sides. The sides of an equilateral polygon have the same length. 
gon	“angle”: A pentagon has five angles. 
hedron	“seat,” “base”: refers to faces of a solid figure. A decahedron is a polyhedron with ten faces.
iso	“equal,” “the same”: An isosceles triangle has two equal sides. Isomorphic objects have the same form. Some folks maintain their equilibrium by doing isometric exercises. “equi” comes from Latin, while “iso” comes from Greek. 
lateral	“side”: A quadrilateral has four sides. A movement sideways is a lateral movement.
ortho	“right angle,” “upright,” “correct”: Two lines are orthogonal if they are perpendicular. Orthorhombic crystals have three axes that come together at right angles. An orthodontist corrects teeth. 
peri	“around,” “about”: The perimeter of a plane figure is the distance around it. A periscope allows one to see around an obstruction.
poly	“many”: A polygon has many (three or more) angles. A polytechnic institute is devoted to instruction in many applied sciences. A polytheist believes in more than one god. A polygraph records several body activities simultaneously.

NUMBER PREFIXES

uni	“one”: The units place is to the left of the decimal. We form the union of sets by unifying them. Some problems have unique solutions. A unicycle has one wheel. A unicorn has one horn.
mono	“alone,” “one”: A monomial is a polynomial with only one term. A monorail has one rail. He delivered his monologue about monogamous relationships in a dull monotone.
bi, bis	“two”: The binary number system has base two. We bisect an angle. Some counties collect property tax biannually, in March and November. Congressional elections are held biennially, in even-numbered years. Man and ape are bipeds.
di, dy	“two”: The angle between two planes is a dihedral angle. A dyadic rational number is the quotient of an integer and a power of two. Physicists deal with dipoles, chemists with dioxides. A diphthong is a gliding speech form with two sounds. “di” comes from Greek, “bi” from Latin.
tri	“three”: A triangle has three sides. A triathlon has three events: cycling, swimming, and running. 
quad	“four”: A quadrilateral is a polygon with four sides. Coordinate axes divide the plane into four quadrants. The Olympic Games are a quadrennial event. 
tetra, tra	“four”: A tetrahedron is a solid figure with four faces. A trapeze is a flying quadrilateral. “tetra” stems from Greek, while “quad” stems from Latin. The “tra” in “trapezoid” and “trapeze” is an abbreviated form of “tetra.”
penta	“five”: A pentagon has five sides. A pentahedron is a solid figure with five faces. The Pentagon Building in Washington, D.C., has five sides. 
hexa	“six”: A hexagon has six sides. The cells in a beehive are hexagonal. 

hepta	“seven”: A heptagon has seven sides. A heptahedron has seven faces. 
octa	“eight”: An octagon has eight sides. The three coordinate planes divide space into eight octants. The musical octave has eight whole notes. An octopus has eight tentacles. 
nona	“nine”: A nonagon has nine sides. A nonagenarian is in his (or, more likely, her) nineties. Another (rarely used) word for a nine-sided polygon is “enneagon.” The prefix “nona” comes from Latin, “ennea” from Greek.
deca	“ten”: A decagon has ten sides. A decade has ten years. A decathlon has ten track-and-field events.
dodeca	“twelve”: A dodecahedron has twelve faces. The regular dodecahedron is one of the five Platonic solids. It is occasionally used for calendars.
icosa	“twenty”: An icosahedron has twenty faces. The regular icosahedron is one of the five Platonic solids. It has twelve vertices, and it can be obtained from a regular dodecahedron by placing a vertex at the center of each face of the dodecahedron.
cent	“hundred”: A centimeter is a hundredth of a meter. A centennial is a hundredth anniversary. A century is a hundred years long.
milli	“thousand”: We have entered a new millennium. A millisecond is a thousandth of a second.

RATIOS AND PROPORTIONAL RELATIONSHIPS

See the *MathLinks*: Grade 7 Resource Guide: Part 1 for additional vocabulary, explanations, and examples for Ratios and Proportional Relationships.

Sense-Making Strategies to Solve Proportional Reasoning Problems

Example 1: How much will 5 pencils cost if 8 pencils cost $4.40?

Strategy 1: Use a "halving" strategy

If 8 pencils cost $4.40, then
4 pencils cost $2.20,
2 pencils cost $1.10, and
1 pencil costs $0.55.

Therefore, 5 pencils cost

$$\$0.55 + \$2.20 = \$2.75.$$

Strategy 2: Find unit prices

First, find the cost of one pencil.

$$\frac{\$4.40}{8} = \$0.55$$

Then, multiply by 5 to find the cost of 5 pencils,

$$(\$0.55)(5) = \$2.75.$$

Example 2: Four square feet of a wall can be painted in 5 minutes. At this same rate, how long will it take to paint the entire 8 foot by 20 foot wall?

Strategy 1: Make a table

First, find the number of square feet in the 8 ft by 20 ft wall.

$$8 \bullet 20 = 160 \text{ sq. ft.}$$

Then, create a table with columns for area and time.

Area	Time
4 sq. ft.	5 minutes
8 sq. ft.	10 minutes
16 sq. ft.	20 minutes
160 sq. ft.	200 minutes

It will take 200 minutes to paint the wall.

Strategy 2: Focus on units

First, find the total number of square feet to be painted.

$$8 \bullet 20 = 160 \text{ sq. ft.}$$

Then, divide by 4 to see how many 5-minute intervals it will take.

$$\frac{160}{4} = 40 \text{ 5-minute intervals}$$

Finally, multiply 40 intervals by 5 minutes per interval to get the total time.

$$40 \bullet 5 = 200 \text{ minutes}$$

It will take 200 minutes to paint the wall.

Sense-Making Strategies to Solve Proportional Reasoning Problems (Continued)

Example 3: Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in $1\frac{1}{2}$ minutes?

Strategy 1: Make a table

Distance	Time
12 ft	3 seconds
4 ft	1 second
240 ft	60 sec = 1 min
120 ft	30 sec = $\frac{1}{2}$ min
360 ft	90 sec = $1\frac{1}{2}$ min

Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Strategy 2: Find unit rates

First, find the rate for 1 second.

$$\frac{12 \text{ ft}}{3 \text{ sec}} = \frac{4 \text{ ft}}{1 \text{ sec}}$$

Then, convert $1\frac{1}{2}$ min to seconds.

$$1\frac{1}{2} \text{ min} = 60 \text{ sec} + 30 \text{ sec} = 90 \text{ sec}$$

Finally, multiply the unit rate by a form of 1.

$$\frac{4 \text{ ft}}{1 \text{ sec}} \bullet \frac{90 \text{ sec}}{90 \text{ sec}} = \frac{360 \text{ ft}}{90 \text{ sec}}$$

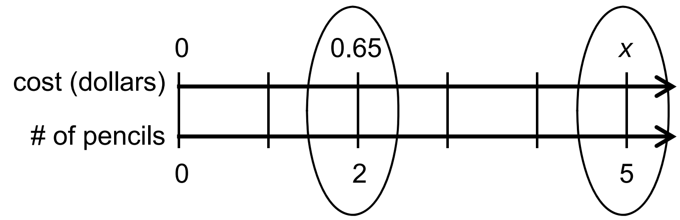
Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Setting Up Proportions

Here are some ways to set up a proportion to solve a problem.

Example: If 2 pencils cost \$0.65, how much will 5 pencils cost?

Strategy 1: Compare rates. This is sometimes referred to as a “between” proportion because the ratios contain different units (i.e., between two units). Between proportions can be read directly from a double number line.



First, use the information in the double number line to create equivalent ratios. (You don’t have to create the double number line first.)

$$\frac{0.65 \text{ dollars}}{2 \text{ pencils}} = \frac{0.65}{2} \ \frac{\text{dollar}}{\text{pencil}} \qquad \frac{x \text{ dollars}}{5 \text{ pencils}} = \frac{x}{5} \ \frac{\text{dollar}}{\text{pencil}}$$

Then, equate the two expressions and solve for x.

$$\frac{x}{5} = \frac{0.65}{2}$$

$x = 1.63$ dollars for 5 pencils.

Note: The equation $\frac{5}{x} = \frac{2}{0.65}$ is another valid “between” proportion for this problem.

Strategy 2: Compare like units. This is sometimes referred to as a “within” proportion because the ratios contain the same units (i.e., within the same unit).

First, create one ratio based on two costs and another ratio based on the corresponding numbers of pencils.

$$\frac{\text{cost}_{\text{case 1}}}{\text{cost}_{\text{case 2}}} = \frac{0.65}{x} \qquad \frac{\text{pencils}_{\text{case 1}}}{\text{pencils}_{\text{case 2}}} = \frac{2}{5}$$

Then, equate the two ratios, and solve for x.

$$\frac{0.65}{x} = \frac{2}{5}$$

$x = 1.63$ dollars for 5 pencils.

Note: The equation $\frac{x}{0.65} = \frac{5}{2}$ is another valid “within” proportion for this problem.

Some Properties Relevant to Solving Proportions

Here are some important properties of arithmetic and equality related to proportions.

- The symmetric property of equality states that if $a = b$, then $b = a$.

  Example: If $30 = 2x$, then $2x = 30$.

- The multiplication property of equality states that equals multiplied by equals are equal. Thus if $a = b$ and $c = d$, then $ac = bd$.

  Example: If $\frac{6}{2} = 3$ and $5 = 9 - 4$, then $\frac{6}{2}(5) = 3(9 - 4)$.

- The multiplication property of 1 (multiplicative identity property) states that

  $$a \bullet 1 = 1 \bullet a = a \text{ for any number } a.$$

  Example: This property is used when writing equivalent fractions, as shown below with the "big one."

  $$\frac{2}{5} \times \frac{12}{12} = \frac{24}{60}$$

  Since division is the inverse of multiplication, and since 1 is its own inverse, we also have the identity $a \div 1 = a$.

  $$\frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$$

- The fraction-inverse property states that if two nonzero fractions are equal, then their inverses are equal. That is, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ $(a \neq 0, b \neq 0, c \neq 0, d \neq 0)$.

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $\frac{7}{5} = \frac{x}{12}$.

Some Properties Relevant to Solving Proportions (Continued)

- The <u>numerator-equality property</u> states that if two equal fractions have equal denominators, then their numerators are equal. Thus if $\frac{a}{b} = \frac{c}{b}$, then $a = c \quad (b \neq 0)$.

  Example: If $\frac{5x}{60} = \frac{85}{60}$, then $5x = 85$.

- The <u>cross-multiplication property</u> states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc \quad (b \neq 0,\ d \neq 0)$.

  This can be remembered with the diagram: $\frac{a}{b} = \frac{c}{d}$.

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $5 \bullet x = 7 \bullet 12$.

  To see that this property is reasonable, try simple numbers:

  If $\frac{3}{4} = \frac{6}{8}$, then $3 \bullet 8 = 4 \bullet 6$.

Applying Properties to Solve Proportions

Strategy 1: The Numerator-Equality Property

Solve for x:

$$\frac{x}{12} = \frac{3}{8}$$

$$\frac{x}{12} \bullet \frac{8}{8} = \frac{3}{8} \bullet \frac{12}{12}$$

$$\frac{8x}{12 \bullet 8} = \frac{36}{8 \bullet 12}$$

Numerator equality property

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

Strategy 2: Cross-Multiplication Property

Solve for x:

$$\frac{x}{12} = \frac{3}{8}$$

Cross multiplication property

$$8 \bullet x = 3 \bullet 12$$

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

A Proof of the Cross-Multiplication Property

The cross-multiplication property is:

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. (Assume $b \neq 0$, $d \neq 0$.)

Statement	Reason
$\frac{a}{c} = \frac{b}{d}$	Given
$a \bullet \frac{1}{b} = c \bullet \frac{1}{d}$	Definition of division
$bd\left(a \bullet \frac{1}{b}\right) = bd\left(c \bullet \frac{1}{d}\right)$	Multiplication property of equality
$ad\left(b \bullet \frac{1}{b}\right) = bc\left(d \bullet \frac{1}{d}\right)$	Commutative/associative properties
$ad \bullet 1 = bc \bullet 1$	Multiplicative inverse property
$ad = bc$	Multiplicative identity property

Another Proof of the Cross-Multiplication Property

This proof of the cross-multiplication property is based on the **numerator-equality property** (which itself requires proving).

Statement	Reason
$\frac{a}{b} = \frac{c}{d}$	Given
$\frac{a}{c} \bullet \frac{d}{d} = \frac{b}{d} \bullet \frac{c}{c}$	Multiplicative identity property
$\frac{ad}{cd} = \frac{bc}{dc}$	Multiplication of fractions
$\frac{ad}{cd} = \frac{bc}{cd}$	Commutative property of multiplication
$ad = bc$	Numerator-equality property

Using Tables to Determine if a Relationship is Proportional

One way to explore if variables are in a proportional relationship is to examine unit rates. If the unit rates for the ratios of values of one variable to the corresponding values of the other variable are all the same, then the one variable is a multiple of the other, and the variables are in a proportional relationship.

Example 1: Rachel bought an annual pass to the amusement park for $60. Then she paid a reduced admission fee of $10 each time she visited the park. This chart shows her total cost for a given number of trips to the amusement park.

# of trips	**0**	**1**	**2**	**3**	**4**
total cost in dollars	60	70	80	90	100
cost per trip in dollars per trip (unit rate)	--	70	40	30	25

Since the costs per trip (unit rates) are not the same, the total cost and the number of trips are not in a proportional relationship.

Example 2: As part of his weekly workout, Nate runs 10 laps around the track. He tries to maintain the same speed for the entire 10 laps. Last week Nate's friend timed his splits (partial times) with the following results:

distance	**1 lap**	**3 laps**	**6 laps**	**10 laps**
time	1 min 45 sec	5 min 15 sec	10 min 30 sec	17 min 30 sec

Is Nate running his laps at the same pace? In other words, is there a proportional relationship between the number of laps Nate runs and Nate's times? To answer this question, we rewrite all times in seconds (the same units) and find the seconds per lap (unit rates).

Number of laps	**1**	**3**	**6**	**10**
seconds	105	315	630	1050
seconds per lap (unit rate)	105	105	105	105

Since the unit rates are all equal, Nate is maintaining the same speed of 105 seconds per lap. This table shows that the number of laps and the running times are in a proportional relationship.

Multiple Representations and Proportional Relationships

Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

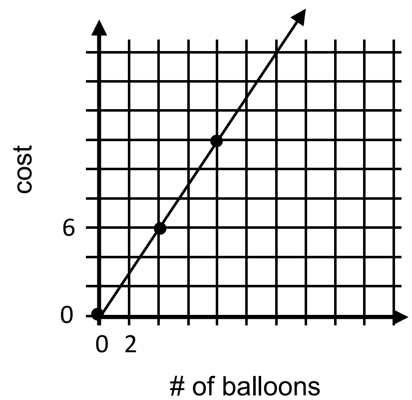
Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Number of Balloons	Cost	Unit Price
4	$6.00	$1.50
2	$3.00	$1.50
1	$1.50	$1.50
8	$12.00	$1.50

Strategy 2: Graphs

A straight line through the origin indicates a proportional relationship.



Strategy 3: Equations

An equation of the form $y = kx$ indicates a proportional relationship. In this case,

y = cost in dollars
x = number of balloons
k = cost per balloon (unit price)

To determine the unit price, create a ratio: $\frac{6 \text{ dollars}}{4 \text{ balloons}} = 1.50 \frac{\text{dollars}}{\text{balloon}}$

Therefore, $k = 1.50$ dollars per balloon, and

$$y = 1.50x.$$

This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

PERCENT

A percent is a number, expressed in terms of the unit 1% = $\frac{1}{100}$. The number *n*% is equal to $\frac{n}{100}$. It is the value of the ratio *n* : 100, and we can think of it as "*n* per 100."

Some Fraction-Decimal-Percent Equivalents		
$\frac{1}{2} = \frac{50}{100} = 0.5 = 50\%$ $\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$ $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$ $\frac{5}{4} = \frac{125}{100} = 1.25 = 125\%$ Conversion strategy: Think: $\frac{3}{4}\left(\frac{25}{25}\right) = \frac{75}{100} = 75\%$	$\frac{1}{10} = \frac{10}{100} = 0.1 = 10\%$ $\frac{3}{10} = \frac{30}{100} = 0.3 = 30\%$ $\frac{5}{10} = \frac{50}{100} = 0.5 = 50\%$ Conversion strategy: Think: $\frac{3}{10} = \frac{30}{100}$ so 0.3 = 0.30 = 30%	$\frac{1}{25} = \frac{4}{100} = 0.4 = 4\%$ $\frac{16}{25} = \frac{64}{100} = 0.64 = 64\%$ $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Conversion strategy: Think: 25(4) = 100, so $\frac{16}{25}\left(\frac{4}{4}\right) = \frac{64}{100} = 64\%$
$\frac{3}{20} = \frac{15}{100} = 0.15 = 15\%$ $\frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$ $\frac{19}{20} = \frac{95}{100} = 0.95 = 95\%$ Conversion strategy: Think: 20 nickels in a dollar $\frac{1}{20}$ of a dollar is \$0.05	$\frac{1}{5} = \frac{2}{10} = 0.2 = 20\%$ $\frac{2}{5} = \frac{4}{10} = 0.4 = 40\%$ $\frac{3}{5} = \frac{6}{10} = 0.6 = 60\%$ $\frac{4}{5} = \frac{8}{10} = 0.8 = 80\%$ Conversion strategy: Think: If I know tenths, I can easily convert to hundredths.	$\frac{1}{8} = \frac{12.5}{100} = 0.125 = 12.5\%$ $\frac{3}{8} = \frac{37.5}{100} = 0.375 = 37.5\%$ $\frac{5}{8} = \frac{62.5}{100} = 0.625 = 62.5\%$ $\frac{7}{8} = \frac{87.5}{100} = 0.875 = 87.5\%$ Conversion strategy: Think: $\frac{1}{4} = \frac{25}{100}$ so half of $\frac{1}{4} = \frac{1}{8} = \frac{12.5}{100} = 12.5\%$

Using Multiplication, Division, and “Chunking” to Find Percents of Numbers

Think	Example
Finding 100% of something is the same as finding all of it.	100% of \$80 = \$80 $\underbrace{\boxed{\quad 100\% \quad}}_{\$80}$
Finding 50% of something is the same as finding one-half of it. This is the same as multiplying by $\frac{1}{2}$ or dividing by 2.	50% of \$80 = $\frac{1}{2}$(\$80) = \$40 \$80 ÷ 2 = \$40 $\underbrace{\boxed{\quad 50\% \quad}\boxed{\quad 50\% \quad}}_{\$80}$
Finding 25% of something is the same as finding one-fourth of it. This is the same as multiplying by $\frac{1}{4}$ or dividing by 4.	25% of \$80 = $\frac{1}{4}$(\$80) = \$20 \$80 ÷ 4 = \$20 $\underbrace{\boxed{25\%}\boxed{25\%}\boxed{25\%}\boxed{25\%}}_{\$80}$
Finding 10% of something is the same as finding one-tenth of it. This is the same as multiplying by $\frac{1}{10}$ or dividing by 10.	10% of \$80 = $\frac{1}{10}$(\$80) = \$8 \$80 ÷ 10 = \$8
Finding 1% of something is the same as finding one-hundredth of it. This is the same as multiplying by $\frac{1}{100}$ or dividing by 100.	1% of \$80 = $\frac{1}{100}$(\$80) = \$0.80 \$80 ÷ 100 = \$0.80
Finding 20% of something is the same as doubling 10% of it.	20% of \$80 = 2(\$8) = \$16
Finding 5% of something is the same halving 10% of it.	5% of \$80 = $\frac{1}{2}$(\$8) = \$4
Finding 15% of something is the same as adding 10% of it and 5% of it.	15% of \$80 = \$8 + \$4 = \$12

Using Multiplication to Find Percents of Numbers

Some percents are hard to find mentally. For example, finding 17% of something is the same as finding $\frac{17}{100}$ = 0.17 of it. In this case, it may be easier to find the percent by using the definition of a percent of a number:

> A percent of a number is the product of the percent and the number.

Example: Find 17% of \$80.

Strategy 1: Use fractions

$$\frac{17}{100} \bullet 80 = \frac{17 \bullet 80}{100} = \frac{1360}{100} = 13.60$$

So 17% of \$80 is \$13.60.

Strategy 2: Use decimals

$(0.17) \bullet (80) = 13.6$

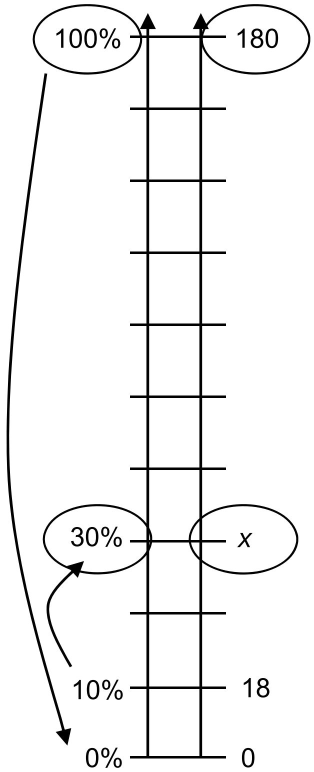
$0.6 = 0.60$

So 17% of \$80 is \$13.60.

Using Double Number Lines to Solve Percent Problems

Strategy 1: Solve on the double number line

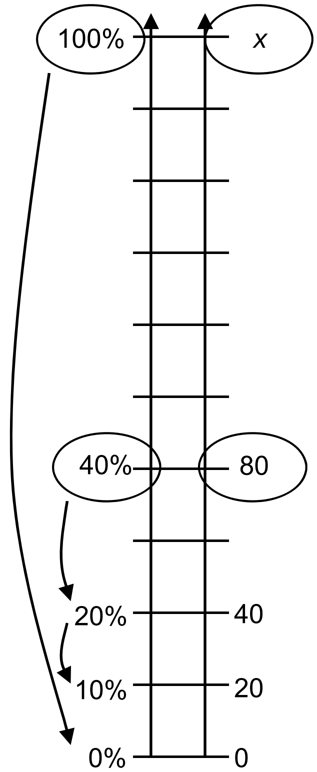
30% of 180 is what amount?



Think: If 100% → 180
then 10% → 18
and 30% → 54

30% of 180 is 54

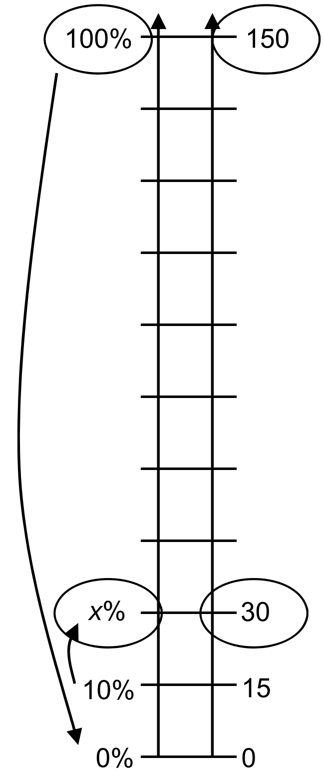
40% of what amount is 80?



Think: If 40% → 80,
then 20% → 40
and 10% → 20
so 100% → 200

40% of 200 is 80

What percent of 150 is 30?



Think: If 100% → 150
then 10% → 15
so 20% → 30

20% of 150 is 30

Strategy 2: Identify equivalent ratios and solve a proportion

30% of 180 is what amount?

$$\frac{100}{30} = \frac{180}{x}$$

or

$$\frac{30}{100} = \frac{x}{180}$$

$x = 54$

30% of 180 is 54

40% of what amount is 80?

$$\frac{100}{40} = \frac{x}{80}$$

or

$$\frac{40}{100} = \frac{80}{x}$$

$x = 200$

40% of 200 is 80

What percent of 150 is 30?

$$\frac{100}{x} = \frac{150}{30}$$

or

$$\frac{x}{100} = \frac{30}{150}$$

$x = 20$

20% of 150 is 30

Percent Increase

Percent increases occur frequently as tips, taxes, and price markups. To find a percent increase, find the amount of the increase and add it to the original quantity.

Example	Original amount	Percent increase	Amount of increase	New amount (original + increase)
Leave a **tip** on a restaurant bill.	\$40	20%	20% of \$40 = \$8	\$40 + \$8 = \$48
Pay **tax** on a clothes purchase.	\$50	8%	8% of \$50 = \$4	\$50 + \$4 = \$54
Pay a **markup** on a video game.	\$75	10%	10% of \$75 = \$7.50	\$75 + \$7.50 = \$82.50

Percent increase also occurs when computing simple annual interest.

Example	Loan amount (principal)	Annual Interest rate	Amount of Interest per year	Total owed after one year (principal + interest)
Obtain a **loan** to buy a car.	\$1,000	4.5%	4.5% of \$1,000 = \$45	\$1,000 + \$45 = \$1,045

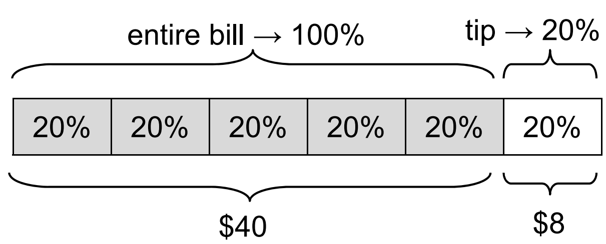
Strategies for Finding Percent Increase

Here are three ways to find percent increase. Consider the tip example above.

How much will you pay if a restaurant bill is \$40 and you leave a 20% tip?

Strategy 1: Make a tape diagram

Since you are paying 100% of the bill and an extra 20% as tip, you are paying 120% of the bill.



Each rectangle of the tape represents \$8. Therefore, the bill plus tip is \$48.

Strategy 2: Set up a proportion

$$\frac{\text{before tip}}{\text{after tip}} \rightarrow \frac{100\%}{120\%} \rightarrow \frac{1}{1.2} = \frac{\$40}{x}$$

$x = 1.2(40)$

bill plus tip → $x = \$48$

Strategy 3: Calculate in one step

120% of \$40 = 1.2(40) = 48

The bill plus tip is \$48.

Percent Decrease

Sales and discounts may be described as percent decreases. To find a percent decrease, find the amount of the decrease and subtract it from the original quantity.

Example	Original amount	Percent decrease	Amount of decrease	New amount (original – decrease)
Sale on shoes purchase	\$50	25%	25% of \$50 = \$12.50	\$50 – \$12.50 = \$37.50
Discount on a dress	\$90	40%	40% of 90 = \$36.00	\$90 – \$36 = \$54

Strategies for Finding Percent Decrease

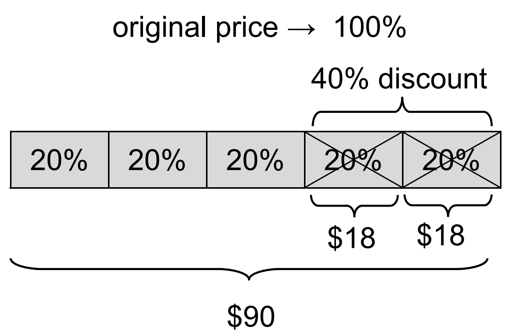
Here are three ways to find percent decrease. Consider the dress discount example above.

A dress costs \$90. How much will you pay if there is a 40% discount?

Strategy 1: Make a tape diagram

A discount of 40% off the original price is the same as paying 60% of the original price.

Each rectangle of the tape represents \$18. Therefore, the price after the discount is \$90 – \$18 – \$18 = \$54.



Strategy 2: Set up a proportion

$$\frac{\text{before discount}}{\text{after discount}} \rightarrow \frac{100\%}{60\%} \rightarrow \frac{1}{0.6} = \frac{\$90}{x}$$

$$x = 0.6(90)$$

Price after discount

$$x = \$54$$

Strategy 3: Calculate in one step

60% of \$90 = 0.6(90) = 54

The price after the discount is \$54.

Sneaky Percent Situations

Sneaky Situation 1:

Have you ever been to a store during a sale and seen a sign like this?

25% OFF +

Take an additional 25% off at the register.

Do you get a total of 50% off? ***NO!***

Suppose the original price of a pair of pants is $100. A 50% savings would lead to a sale price of $50.

However, the calculation for the sale above is different.

Original savings →	25% of $100 = $25
New amount →	$75
Additional savings →	25% of $75 = $18.75
Final sale price →	$56.25

$56.25 is a greater price than $50.

Sneaky Situation 2:

A Yogi Berra baseball card is worth $100 in December. It decreases in value by 20% in January, and then it increases in value by 20% in February.

Is the card back to its original value? ***NO!***

Value in December →	$100
Value in January →	20% of $100 = $20 $100 – $20 = $80
Value in February →	20% of $80 = $16 $80 + 16 = $96

$96 is less than the original value of $100.

Simple vs. Compound Interest

Simple interest calculations (based on the original principal only) are used primarily for loans over short time periods, or for personal loans. However, most financial transactions, such as savings account deposits at banks, are computed using compound interest (based on the original principal and accumulated interest).

<table>
<tr><th>Simple Interest Example</th><th>Compound Interest Example
(Compounded Annually)</th></tr>
<tr><td>You make a two-year loan of $600 to a friend at a simple annual interest rate of 5%. How much principal and interest will you get back after the two years?

Let I = interest
Let P = principal ($600)
Let R = rate (5%)
Let T = time (2 years)
Let A = total amount

At the end of the year 2:

$I = PRT$ $A = P + I$
$I = 600(0.05)(2)$ $A = 600 + 60$
$I = \$60$ $A = \$660$</td><td>You deposit $600 at a bank at an interest rate of 5%, compounded annually. How much principal and interest will you get back after 2 years?

Let I = interest
Let P = principal ($600)
Let R = rate (5%)
Let T = time (2 years)
Let A = total amount

At the end of year 1:

$I = PRT$ $A = P + I$
$I = 600(0.05)(1)$ $A = 600 + 30$
$I = \$60$ $A = \$630$

Principal at the end of year 1 = $630

At the end of year 2:

$I = PRT$ A (year 2) = $P + I$
$I = 630(0.05)(1)$ A (year 2) = 630 + 31.50
$I = \$31.50$ A (year 2) = $661.50</td></tr>
</table>

Therefore you earn $60 in interest over two years with a simple annual interest rate of 5%, while you earn $61.50 ($30 + $31.50) in interest over two years with a 5% interest rate compounded annually.

STATISTICS AND DATA DISPLAYS

Statistics and Statistical Questions

Statistics is the study of the collection, organization, analysis, interpretation, and presentation of data. It helps us answer questions when some sort of variability is anticipated in the population being studied. It provides us with tools to study variable populations, and with measures of center and spread (variability) that may be used to summarize numerical data sets.

A statistical question is one that can be answered by collecting data for which it is anticipated that the data will be variable.

Example of a statistical question: “How much TV do students in my class watch?” This question anticipates variability in the number of hours spent watching TV.

NOT a good statistical question: “How many hours of TV did Albert watch last week?” This question has only one value as an answer.

Sampling

Sampling refers to selecting a subset of a population to be examined for the purpose of drawing statistical inferences about the entire population. If the sample is representative of the entire population, we may make valid inferences about the entire population based on properties of the sample.

Suppose you want to know how many hours per week students in our school spend watching television. From the population of all students, you select a sample and you ask the students in the sample how many hours they watch television. You would like to infer that the average time spent watching TV for all students is about the same as for students in the sample.

An easy way to select a sample might be to ask your friends how many hours they watch TV. Such a sample is called a convenience sample. However, your friends may not be representative of all students.

To select a more representative sample, you might place the names of all students in the school in a hat, mix the names thoroughly, and draw a certain number of names from the hat. This sort of sample is referred to as a random sample. Its mathematical properties allow us to draw inferences about the population.

Dot Plots (Line Plots)

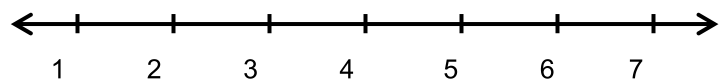
A dot plot (also called a line plot) displays data on a number line with a dot (•) or an X to show the frequency of data values.

Here are the number of siblings (brothers and sisters) for 13 different students:

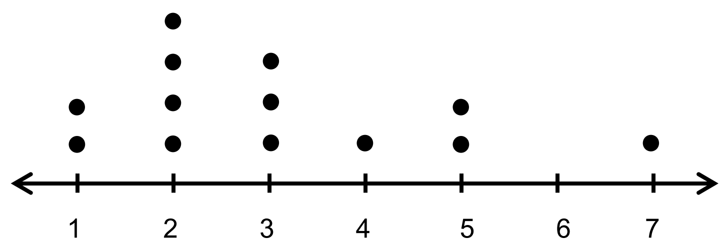
3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

To make a dot plot of this data set:

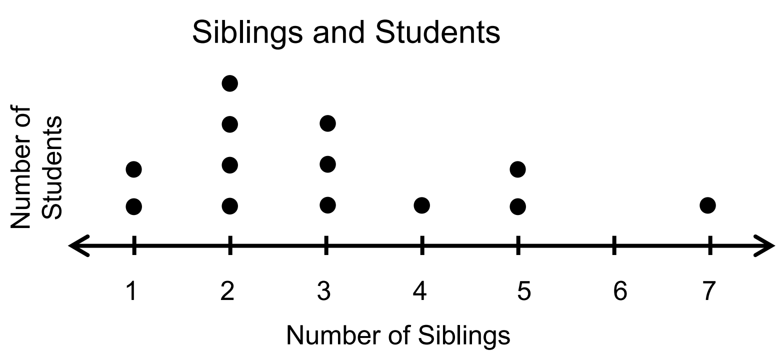
1. Make a number line that extends from the minimum data value to the maximum data value.



2. Mark a dot or an X for every data value.



3. Write a title and add vertical and horizontal labels.



Measures of Center

Here are the number of siblings for 13 different students:

3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

To find the <u>mean</u> (average) of a data set, add all the values in the data set and divide it by the number of values (number of observations, n).

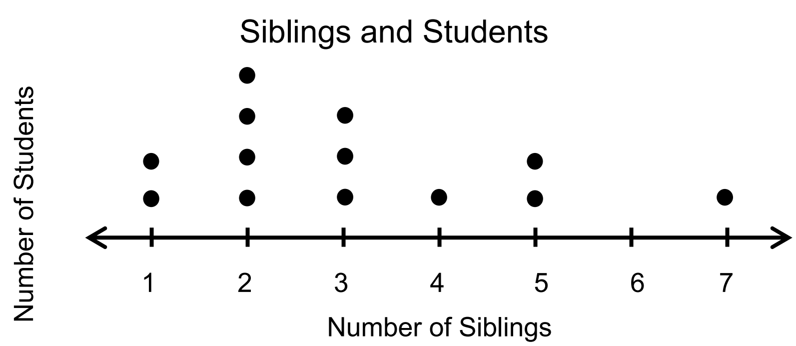
Number of observations: $n = 13$
To find the mean: $3 + 4 + 5 + 2 + 2 + 3 + 3 + 2 + 2 + 5 + 7 + 1 + 1 = 40$
$40 \div 13 \approx 3.08$

To find the <u>median</u> (M), order the values from least to greatest and find the middle number. If there is an even number of values in the data set, the median is the mean (average) of the two middle numbers.

For the siblings data set: {1, 1, 2, 2, 2, 2, 3, 3, 3, 4, 5, 5, 7}
median

To find the <u>mode</u>, find the value that occurs most often. (Some data sets may have more than one mode.)

For the siblings data set, the mode is 2: The value 2 occurs most often, as illustrated in the dot plot to the right.



The Range, the Quartiles, and the Five-Number Summary

Here are the number of siblings for 13 different students:

3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

To find the range of a data set, find the difference between the greatest value and the least value in the data set.

For the siblings data set, the range is 6, since 7 – 1 = 6.

To find quartiles, first put the numbers in numerical order. Then locate the points that divide the data set into four equal parts.

1	1	2	2	2	2	3	3	3	4	5	5	7

min Q_1 $Q_2 = M$ median Q_3 *max*

For the siblings data set: $Q_1 = 2$ (the 1st quartile)
$Q_2 = 3$ (the 2nd quartile; also the median)
$Q_3 = 4.5$ (the 3rd quartile)

Q_1 is the median of the first half of the data set, and Q_3 is the median of the second half.

The five-number summary is $(min, Q_1, Q_2, Q_3, max)$ = (1 , 2 , 3 , 4.5 , 7)

Using Paper Strips to Find the Five-Number Summary

Suppose these numbers represent the number of siblings for 13 different students.

3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

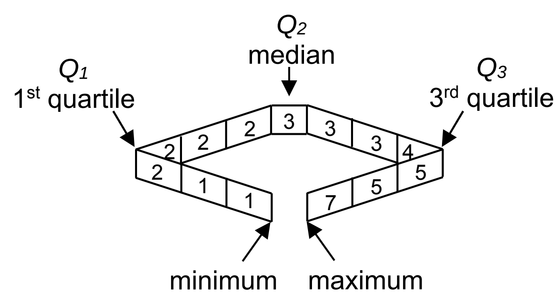
To find the five-number summary using a strip of paper:

1. Enter the numbers, in numerical order, on a blank strip

1	1	2	2	2	2	3	3	3	4	5	5	7

2. Fold the strip as shown below to locate the minimum, 1st quartile (Q_1), median (Q_2), 3rd quartile (Q_3), and maximum.

statistic	Class siblings
Minimum (*min*)	1
1st Quartile (Q_1)	2
Median ($Q_2 = M$)	3
3rd Quartile (Q_3)	4.5
Maximum (*max*)	7



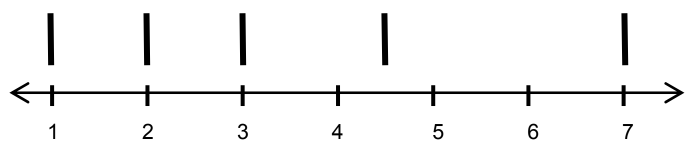
Box Plots (Box-and-Whisker Plots)

A box plot (or box-and-whisker plot) provides a visual representation of the center and spread of a data set. The display is based on the five-number summary.

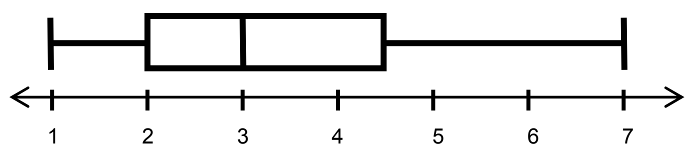
For the sibling data, the five-number summary is (1 , 2 , 3 , 4.5 , 7)

To make a box plot:

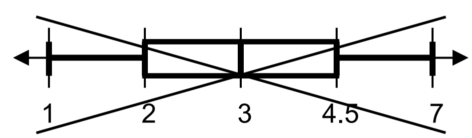
- Locate the five-number summary values on a number line, and indicate with vertical segments.



- Create a “box” to highlight the interval from the first to the third quartile, and draw “whiskers” that extend to the minimum and maximum.



Be sure to scale the box plot properly. This plot is WRONG:



To interpret a box plot:

- Each of the four “sections” (the two whiskers and the two rectangular parts of the box) contains (close to) one-fourth of the data points. Be careful: If one section appears larger than another, we cannot say it has more data points, but only that the data points are spread out over a wider range.

- We have used the word “quartile” to refer to specific data points. Sometimes the word “quartile” is also used to refer to one of the four quarters, or sections, of the data set. For example, data points that lie within the farthest left section may be referred to as “in the first quartile.”

Mean Absolute Deviation

The mean absolute deviation (MAD) is a measure of spread of a numerical data set. It is the arithmetic average of the distance (absolute value) of each data point to the mean. To calculate the MAD statistic:

For the sibling data, there are 13 data points:

3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

To find the MAD statistic:

- Find the mean of the sample.

  The mean is 3.08.

- Find the distance (absolute value) from each data point to the mean.

  See the table entries to the right.

- Find the sum of the distances.

  See the bottom row of the table.

- Divide the sum of the distances by the number of data points to find the average distance from the mean.

  See the calculation at the bottom of page.

Sibling Data	**Distance from data point to mean**
3	$\lvert 3.08 - 3 \rvert = 0.08$
4	$\lvert 3.08 - 4 \rvert = 0.92$
5	$\lvert 3.08 - 5 \rvert = 1.92$
2	$\lvert 3.08 - 2 \rvert = 1.08$
2	$\lvert 3.08 - 2 \rvert = 1.08$
3	$\lvert 3.08 - 3 \rvert = 0.08$
3	$\lvert 3.08 - 3 \rvert = 0.08$
2	$\lvert 3.08 - 2 \rvert = 1.08$
2	$\lvert 3.08 - 2 \rvert = 1.08$
5	$\lvert 3.08 - 5 \rvert = 1.92$
7	$\lvert 3.08 - 7 \rvert = 3.92$
1	$\lvert 3.08 - 1 \rvert = 2.08$
1	$\lvert 3.08 - 1 \rvert = 2.08$
Sum of distances from mean	17.4

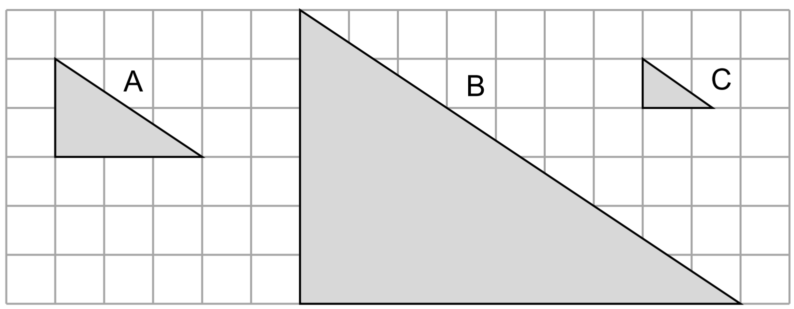
$$\text{MAD} = \frac{\text{sum of distances from mean}}{\text{number of data points}} = \frac{17.4}{13} = 1.34$$

SCALE DRAWINGS

Scale Factors

A scale factor is a positive number which multiplies some quantity. If the scale factor is greater than 1, the figure is expanded, and if the scale factor is between 0 and 1, the figure is reduced in size.

- To make Triangle B below, multiply each dimension of Triangle A by a scale factor of 3. Triangle B is a 300% enlargement of Triangle A.
- To make Triangle C below, multiply each dimension of Triangle A by a scale factor of $\frac{1}{2}$. Triangle C is a 50% reduction of triangle A.



Scale Drawings

A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor.

The flag of Italy is composed of three stripes (green, white, and red) that divide the flag into thirds. Pictured below is a scale drawing of the flag.

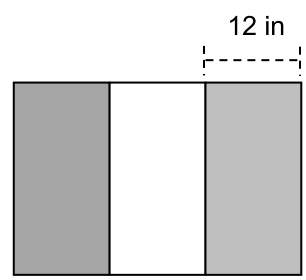
Suppose the original flag is 36 inches by 24 inches, and the scale drawing is 1.5 inches by 1 inch.

This scale may be represented as a ratio:

1.5 inch : 36 inches

1 inch : 24 inches

1 : 24



The scale drawing is a reduction of the flag. The scale factor (value of the ratio) that produces this reduction is $\frac{1}{24}$.

PLANE FIGURES

Symbols and Conventions for Geometry Notation

Below are some geometry notations we will use. Note that we use absolute values to denote lengths of segments and measures of angles. This is consistent with more advanced levels of mathematics.

Points are named by capital letters.

The line segment from P to Q is denoted by $\overline{PQ}$.

The length of the line segment from P to Q is denoted by $|PQ|$, which is shorthand for $|\overline{PQ}|$.

The symbol for triangle is Δ.

- The triangle in Figure 1 below may be denoted by ΔLMN, or also by ΔLNM. Vertices may be listed in either a clockwise or counterclockwise direction starting from any of the three vertices.

The symbol for angle is $\angle$.

- The angle at the top of Figure 1 below can be denoted by $\angle NLM$, or by $\angle 1$, or by $\angle L$.
- The pair of adjacent angles in Figure 2 below are $\angle FGJ$ and $\angle HGF$. They share the common ray $\overrightarrow{GF}$. The two adjacent angles together form the angle $\angle JGH$.

  Error alert: Using "$\angle G$" to name the angle below is ambiguous. We do not know if it refers to $\angle JGF$, $\angle FGH$, or $\angle JGH$.

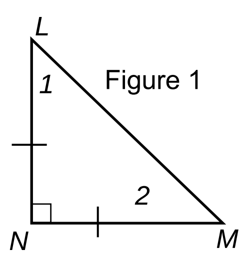
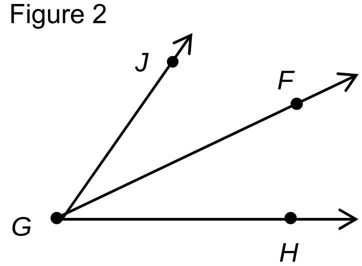


Figure 1

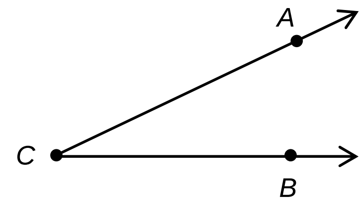
Figure 2


The measure of an angle $\angle A$ is denoted by $|\angle A|$. The small square at N indicates that $\angle LNM$ is a right angle, that is, that $|\angle LMN| = 90°$.

The single hash marks on the segments $\overline{LN}$ and $\overline{NM}$ indicate that the segments have equal length, that is, $|LN| = |NM|$.

Classifying Angles By Their Degree Measure

An angle is a geometric shape formed by two (distinct) rays that share a common endpoint (the vertex of the angle).



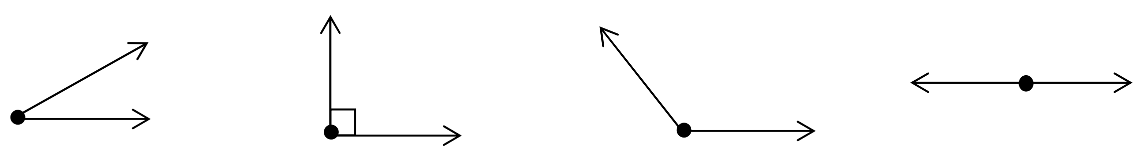
The angle in the figure above can be named any one of the following:

$\angle ACB$ or $\angle BCA$ or $\angle C$

The point C is the vertex of the angle. The rays $\overrightarrow{CA}$ and $\overrightarrow{CB}$ meet at C and form the sides of the angle.

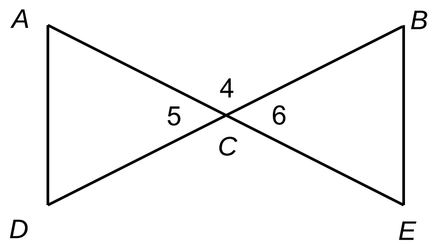
To each angle is assigned a degree measure between 0 and 180 degrees, which indicates the size of the angle. Angles may be classified by their degree measure.

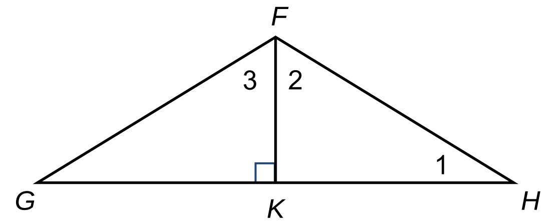
- An acute angle is an angle whose measure is less than 90°.
- A right angle is an angle whose measure is exactly 90°.
- An obtuse angle is an angle whose measure is between 90° and 180°.
- A straight angle is an angle whose measure is 180°. The sides of a straight angle are opposite rays that form a straight line.



acute angle right angle obtuse angle straight angle

Special Angle Pairs





Angle Pairs	Defining Properties	Examples
complementary angles	sum of degree measures is 90°	$\angle KHF$ and $\angle KFH$ ($\angle 1$ and $\angle 2$)
supplementary angles	sum of degree measures is 180°	$\angle ACB$ and $\angle BCE$ ($\angle 4$ and $\angle 6$)
adjacent angles	two angles that share a common vertex and ray, and lie on opposite sides of the ray	$\angle GFK$ and $\angle KFH$ ($\angle 3$ and $\angle 2$)
vertical angles	opposite angles formed when two lines intersect	$\angle ACD$ and $\angle BCE$ ($\angle 5$ and $\angle 6$)

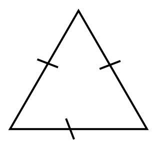
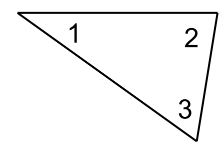
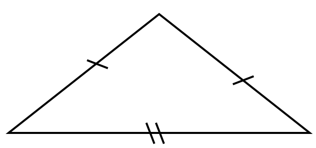
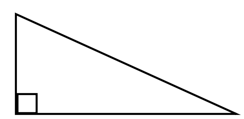
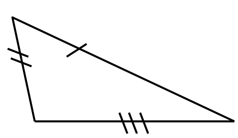
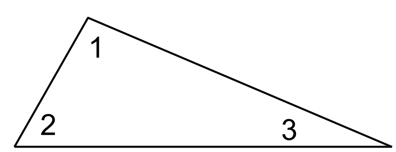
<u>Some facts about angles</u>:

Any two right angles are supplementary. This is because a right angle measures 90°, so any two right angles have measures adding up to 180°.

In a right triangle, the two lesser angles are always complementary. This is because the sum of the measures of the angles of a triangle is 180°. Since the right angle measures 90°, the sum of the other two angles must be 90°.

Classifying Triangles

A triangle is a three-sided polygon. Triangles may be classified by their sides or by their angles.

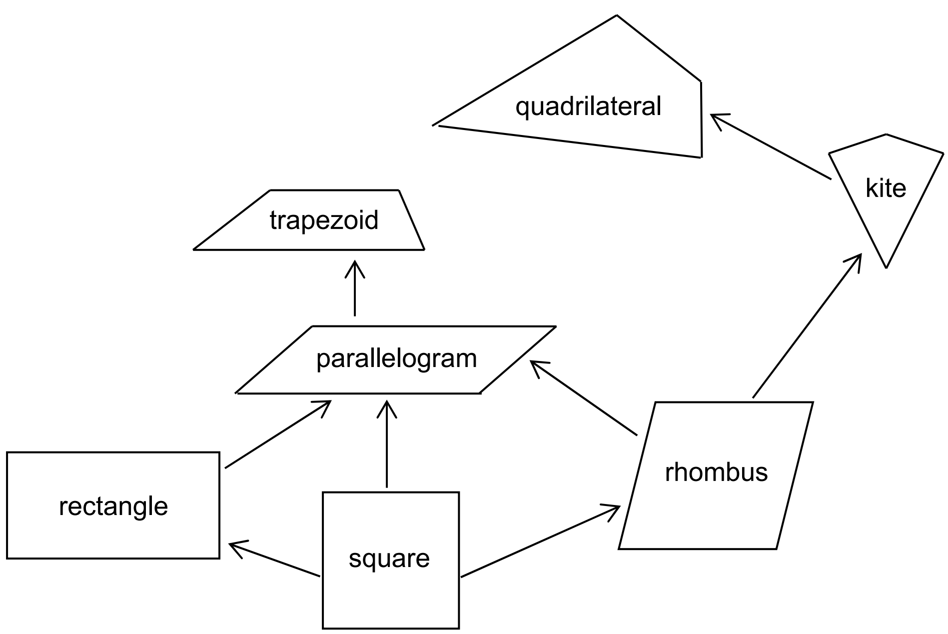
Classification by Sides	Classification by Angles
An <u>equilateral</u> triangle is a triangle with three congruent sides. 	An <u>acute</u> triangle is a triangle with three acute angles.  $\lvert\angle 1\rvert < 90°$ $\lvert\angle 2\rvert < 90°$ $\lvert\angle 3\rvert < 90°$
An <u>isosceles</u> triangle is a triangle with at least two congruent sides. 	A <u>right</u> triangle is a triangle with one right angle.  The square in the corner indicates that the angle measures 90°.
A <u>scalene</u> triangle is a triangle with no congruent sides. 	An <u>obtuse</u> triangle is a triangle with one obtuse angle.  $\lvert\angle 1\rvert > 90°$ $\lvert\angle 2\rvert < 90°$ $\lvert\angle 3\rvert < 90°$

Note that an equilateral triangle is also <u>equiangular</u> because all three angles measure 60°.

Some Properties of Quadrilaterals

A quadrilateral is a four-sided polygon. Some of the common types of quadrilaterals are:

rectangle	A quadrilateral with four right angles. Opposite sides of a rectangle are parallel and have the same length.
square	A quadrilateral with four congruent sides and four right angles. A square is a rectangle.
parallelogram	A quadrilateral in which opposite sides are parallel. Opposite sides of a parallelogram have the same length, and opposite angles have the same measure.
rhombus	A quadrilateral whose four sides have the same length. A square is a rhombus, but a rhombus is not necessarily a square. (The plural of "rhombus" is either "rhombuses" or "rhombi.")
trapezoid	A quadrilateral with at least one pair of parallel sides.
kite	A quadrilateral whose four sides can be grouped in two pairs of adjacent sides of the same length. The two vertices where the congruent sides meet determine a line of symmetry of the kite.



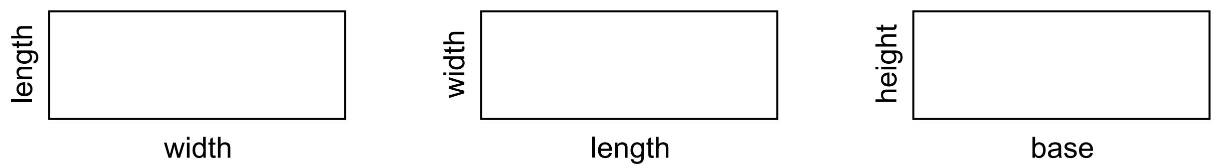
Which Side of a Rectangle is its Base?

We may select any side of the rectangle and refer to it as the base. In the formula,

$$\text{Area of rectangle} = (\text{length}) \times (\text{width}) = \ell w,$$

the "length" then refers to the length of the base, while the "width" refers to the length of the sides perpendicular to the base. The width is the same as the height of the rectangle, and the formula for the area can also be written

$$\text{Area of rectangle} = (\text{length of base}) \times (\text{height}) = bh.$$

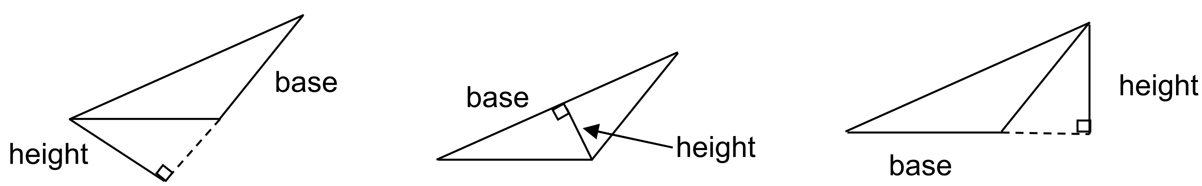


Which Side of a Triangle is its Base?

We may select any side of a triangle and declare it to be the base of the triangle. The height of the triangle is the perpendicular distance from the vertex opposite the base to the base (extended if necessary). The formula for the area of the triangle is

$$\text{Area of triangle} = \frac{1}{2}(\text{length of base}) \times (\text{height}) = \frac{1}{2}bh.$$

The values we substitute into the area formula depend on which side we select as base. The three different choices of base may lead to three different calculations for the area of a triangle, corresponding to the three heights and lengths of base. Of course, the three calculations give the same result.

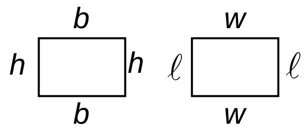
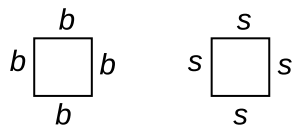
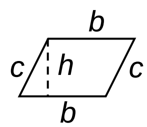
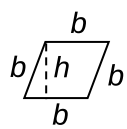
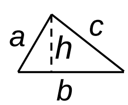
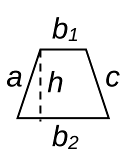
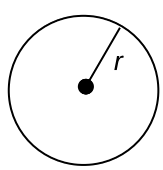


About Pi

Pi (also written as the Greek letter π) is the value of the ratio of the circumference of a circle to its diameter. The constant π is slightly greater than 3, so that the circumference of a circle is a little more than 3 times its diameter.

Though we often use 3.14 or $\frac{22}{7}$ for the value of π, these are only approximations. It can be shown that π is not a rational number. That is, pi cannot be represented as a quotient of two integers. The decimal expansion of pi is nonrepeating.

$$\pi = 3.14159265358979323846264338327950288419 71\ldots$$

Summary of Perimeter and Area Formulas			
Shape/Definition	**Diagram**	**Perimeter or Circumference**	**Area**
Rectangle a quadrilateral with 4 right angles		$P = 2b + 2h$ or $P = 2\ell + 2w$	$A = bh$ or $A = \ell w$
Square a rectangle with 4 equal sides		$P = 4b$ or $P = 4s$	$A = b^2$ or $A = s^2$
Parallelogram a quadrilateral with opposite sides parallel		$P = 2(b + c)$ or $P = 2b + 2c$	$A = bh$
Rhombus a quadrilateral with 4 equal sides		$P = 4b$	$A = bh$
Triangle a polygon with three sides		$P = a + b + c$	$A = \frac{1}{2}bh$
Trapezoid a quadrilateral with at least one pair of parallel sides		$P = a + b_1 + b_2 + c$	$A = \frac{1}{2}(b_1 + b_2)h$
Circle closed figure in a plane where all points are a fixed distance (radius) from a given point (center)		$C = 2\pi r$	$A = \pi r^2$
For consistency, we illustrate all formulas using *b* to refer to the length of a base. The consistent use of *b* makes the relationships among formulas more apparent.			

Finding Lengths and Areas of Figures

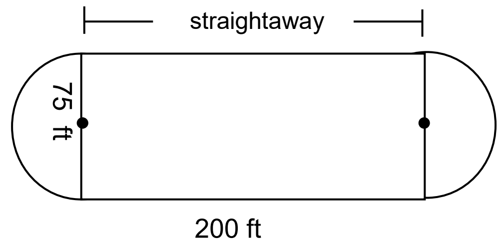
A field at a local school is surrounded by a track. The straightaways are each 200 feet. The diameter of each semicircular region at each end is 75 feet. Find the area of the field and the distance around the track.

Define the variables:

Let ℓ = length straightaway = 200 ft

Let w = height of the rectangle (diameter) = 75 ft

Let r = radius of the circle = 37.5 ft



The area of the field is the sum of the areas of two semicircles and one rectangle.

To find the area A of the field:

Write a formula and substitute:

$$A = \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 + \ell w$$

$$A = \pi r^2 + \ell w$$

$$A \approx 3.14(37.5)^2 + (200)(75)$$

$$A \approx 4{,}415.6 + 15{,}000$$

$$A \approx 19{,}415.6$$

The area of the field is (approximately) 19,415.6 sq. ft.

The distance around the track is the sum of the lengths of two semicircular ends and two straightaways.

To find the distance D around the track:

Write a formula and substitute:

$$D = 2\pi r + 2\ell$$

$$D \approx 2(3.14)(37.5) + 2(200)$$

$$D \approx 235.5 + 400$$

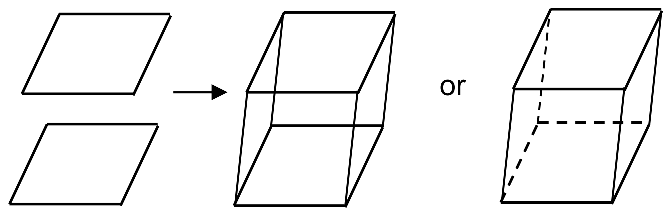
$$D \approx 635.5$$

The distance around the track is (approximately) 635.5 ft.

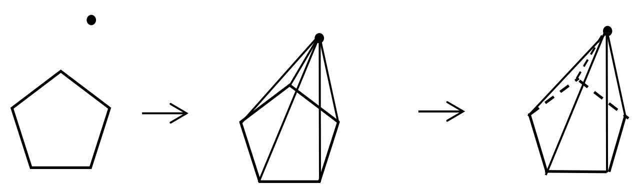
SOLID FIGURES

Drawing Three-Dimensional Figures

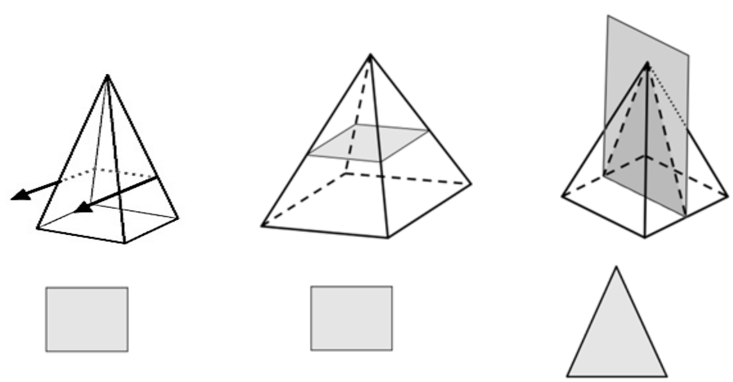
To draw a prism, start by drawing two congruent polygons for the bases. Then connect vertices to complete the figure. You may also use dotted lines to indicate edges that cannot be seen.



To draw a pyramid, start by drawing one polygon for the base. Then put a point somewhere on the paper to represent the apex of the pyramid. Connect the apex to the vertices of the polygon. Redraw using dotted lines to indicate edges you cannot see.



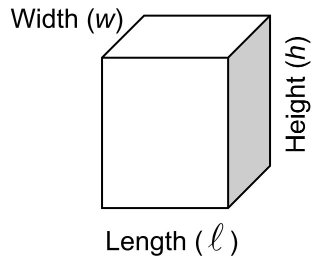
The intersection of a solid figure with a plane is a cross section of the figure. Here are three ways to show a cross section of a pyramid.

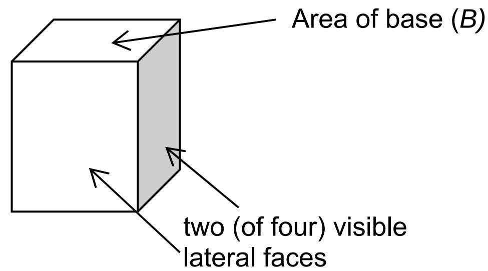


Right Rectangular Prisms

A <u>right rectangular prism</u> (a box) is a six-sided polyhedron in which all the faces are rectangles. The opposite faces of a right rectangular prism are parallel to each other. The distances between pairs of opposite faces are the <u>length</u>, <u>width</u>, and <u>height</u> of the right rectangular prism.

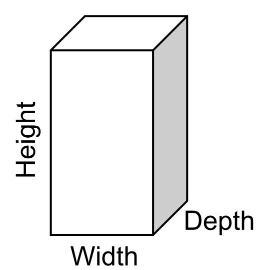
Any two pairs of opposite rectangular faces can be chosen as the bases of the prism. The other rectangular faces are the <u>lateral faces</u>.



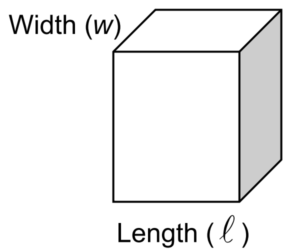


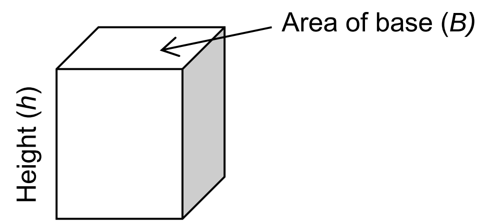
Note that these three words (length, width, and height) all refer to the <u>length</u> of some edge and can sometimes be used interchangably, depending upon context.

Take, for example, a refrigerator, which is much like a right rectangular prism. It is customary for the dimensions to be given as height, width, and depth, as pictured to the right.

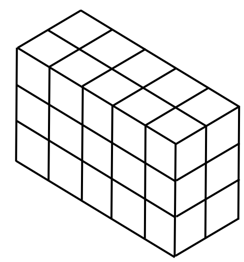


Volume of Right Rectangular Prisms





The area of the base is the product of the length and width ($B = \ell w$).



The <u>volume</u> (V) of a prism may be computed by counting layers of unit cubes. In the prism to the right, each layer has 10 cubes. There are 3 layers.

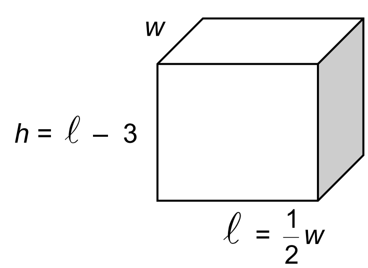
V = (10)(3) = 30 cubic units.

In general, to find the volume, multiply the area of the base (B) by the height of the prism.

$V = \ell wh$ or $V = Bh$

Example: The width of a right rectangular prism is 20 centimeters. The length is half as long as the width. The height is 3 centimeters less than the length. Find its volume and surface area.

Define variables:



Width: w = 20 cm

Length: $\ell = \frac{1}{2}w$ = 10 cm

Height: $h = \ell - 3 = 10 - 3 = 7$ cm

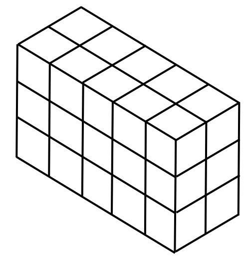
Write a formula and substitute:

$$
\begin{aligned}
V &= \ell wh \\
&= 10 \bullet 20 \bullet 7 \\
&= 1{,}400
\end{aligned}
$$

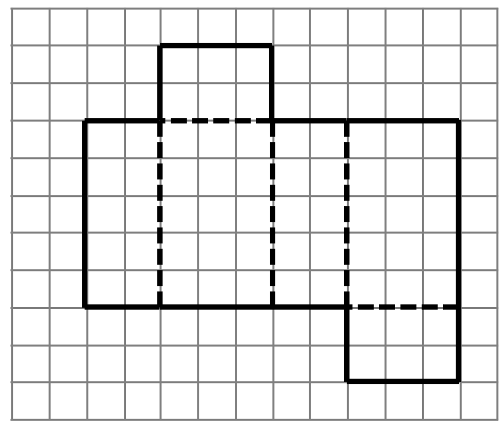
The volume is 1,400 cubic centimeters (1,400 cm^3).

Surface Area of Right Rectangular Prisms

The <u>surface area</u> (*SA*) of a prism may be computed by finding the sum of the areas of all of the faces.



It may be helpful to draw each face separately, or to create a <u>net</u> that shows the areas of each face of the prism. In the prism to the right, there are two faces with dimensions 2×5, two faces with dimensions 3×2, and two faces with dimensions 3×5.



$$SA = 2(2 \times 5) + 2(3 \times 2) + 2(3 \times 5)$$

$$= 20 + 12 + 30 = 62 \text{ square units.}$$

In general, find the area of each rectangular face.

$$SA = \ell w + \ell w + wh + wh + \ell h + \ell h$$

$$SA = 2\ell w + 2wh + 2\ell h$$

$$SA = 2(\ell w + wh + \ell h)$$

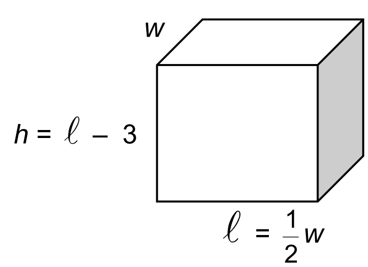
Example: The width of the right rectangular prism below is 20 centimeters. The length is half as long as the width. The height is 3 centimeters less than the length. Find its surface area.

Define variables:

Width: $w = 20$ cm

Length: $\ell = \frac{1}{2}w = 10$ cm

Height: $h = \ell - 3 = 10 - 3 = 7$ cm



Write a formula and substitute:

$$\begin{aligned} \text{SA} &= 2(\ell w + wh + \ell h) \\ &= 2(10 \bullet 20 + 20 \bullet 7 + 10 \bullet 7) \\ &= 2(200 + 140 + 70) \\ &= 2(410) \\ &= 820 \end{aligned}$$

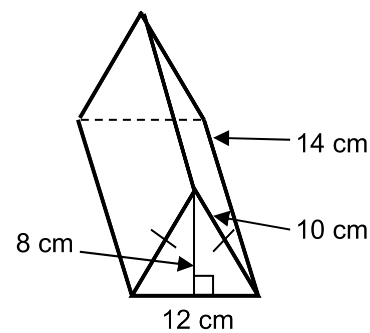
The surface area is 820 square centimeters (820 cm^2).

Other Right Prisms

Every right prism has two faces (the bases) that are congruent parallel polygons, and lateral faces that are rectangles.

Pictured below is a right triangular prism. It has two congruent parallel triangular bases and three faces that are rectangles. It is sitting on one of its lateral faces.

The height of the prism is the distance from one base to the other.



Problem: Find its surface area and volume.

Solution Step 1: Define variables.

Let b = length of triangular base = 12 cm

Let h = height of triangular base = 8 cm

Let H = height of right prism = 14 cm

Step 2: Find the volume.

To find the volume (V) of any right prism, multiply the area of the base (B) by the height (H).

$$V = BH$$

$$V = \left(\frac{1}{2}bh\right)H$$

$$V = \left(\frac{1}{2} \bullet 12 \bullet 8\right) \bullet 14$$

$$V = (48)(14) = 672 \text{ cm}^3$$

Step 3: Find the surface area.

To find the surface area (SA) of any right prism, add the areas of the faces.

Find the area of the triangular base (there are two of these):

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \bullet 12 \bullet 8 = 48 \text{ cm}^2$$

Find the area of the 10 cm × 14 cm rectangular face (there are two of these):

$$A = \ell w = 10 \bullet 14 = 140 \text{ cm}^2$$

Find the area of the 12 cm × 14 cm rectangular face (there is one of these):

$$A = \ell w = 12 \bullet 14 = 168 \text{ cm}^2$$

Finally add areas of the faces.

$$SA = 48 + 48 + 140 + 140 + 168$$

$$SA = 544 \text{ cm}^2$$

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